STUDY OF ATEST PROCEDURE USING SOME PRELIMNARY TESTS OF SIGNIFICANCE IN ACOMPONFNT OF VARIANCE MODEL

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INTRODUCTION

Paull [5] considered the analysis of variance of a two way classification witn both factors random and cell repetition and developed a test procedure known as «sometimes pool test procedure» (SPTP) based on a preliminary tests of significance for testing the hypothesis of no treatment effects. Bozivich, Bancroft and Hartley (1) , (2) extended the study of Paull by deriving more general formulas for size and power which were applicable to any combination of even values of degrres of freedom. Srivastava (6) , (5), Srivastava and Bozivich (7) considered a three-fold nested classification model with random effects and studied the size and power of SPTP based on two preliminary tests of significance. In the present investigation we have concidered the analysis of variance of a four-fold nested classification with all factors random as considered by Graybill (3) and have developed a SPTP based on preliminary tests of significance for testing the hypothesis of no treatment effects. The power of SPTP has been compared with the power of the never pool test (NPT) of the same SPTP has been compared with the power of the never pool test (NPT) of the same size for various combinations of degrees of freedom. On the basis of this comparison we have attempted recommendations on the advisbility or otherwise of using it.

STATMENT OF THE PROBLEM

Suppose we are interested in the study of variability of soil of a State. The State under study comprises of a large number of Revenue Tehsils. A random sample of I Tehsils is drawn from the State and a random sample of J villages is taken from each of I Tehsils. From each of J villages a random sample of K fields is taken and a random sample M sections (pkts) of land is drawn from each of K fields. finally, a random sample of size N required quantity of soil from each of M sections is taken and the amounts of calcium, Potash and organic matters were determined for IJKMN samples. Let y_{ijKmn} denotes the percentagle amount of any substance on nth sample of soil from mth section of the kth fied in jth village of the ith Tehsil. Then the sample observations y_{ijRmu} can well be represented by a balanced four fold nested classification sample model

$$(1)\quad y_{ijkmn} = \mu + a_i + b_{ij} + c_{ijk} + d_{ijkm} + e_{ijkmn}, \quad \begin{array}{l} i = 1, 2, \ldots, I \\ j = 1, 2, \ldots, J \\ k = 1, 2, \ldots, K \\ m = 1, 2, \ldots, M \\ n = 1, 2, \ldots, N \end{array}$$

where the «Tehsil variable» a_i, the «village variable» b_{ij}, the «field variable» c_{ijk}, the «section variable» d_{ijKmn} and the «test variable» e_{ijRmn} are assumed to be random samples from the respective normal population $N(0, \sigma_a^2)$, $N(0, \sigma_b^2)$, $N(0, \sigma_c^2)$ $N(0, \sigma_d^2)$, and $N(0, \sigma_e^2)$. The analysis of variance resulting from model (1) is given in Table I.

TABLE I

Analysis of variance. Fourfold Nested Classification for a component of Variance Model.

Source of Variation	Degrees of Freedom	Mean Squares	Expected Mean Squars
Between Tehsils (Treatments)	$n_5 = I-1$	V_5	$\sigma_5^p = \sigma_4^2 + JKMN\,\sigma_a^2$
Between Villages)True Error)	$n_4 = I(J-1)$	V_4	$\sigma_2^2 = \sigma_3^2 + KMN\,\sigma_b^2$
Between Fields (Doubtful Errors3)	$n_3 = IJ(K-1)$	V_3	$\sigma_3^2 = \sigma_2^2 + MN\,\sigma_c^2$
Between Sections (Doubtful Errors2)	$n_2 = IJK(M-1)$	V_2	$\sigma_2^2 = \sigma_1^2 + N\,\sigma_d^2$
Within Sections (Doubtful Error1)	$n_1 = IJKM(N-1)$	V_1	$\sigma_1^2 = \sigma_e^2$

The five sums squares n_i V_i ($i = 1,2, \ldots 5$) are independently distributed as x_i^2 σ_i^2, where x_i^2 is the central chi-sqare statistic based on n_i degrees of freedom The main interest of the experiment lies in testing the hypethesis of no Tehsit effect, i.e. testing the main hypothesis $H_o : \sigma_5^9 = \sigma_4^2$ ($\sigma_a^2 = \sigma$) against $H : \sigma_4^2 > \sigma_5^2$ ($\sigma_a^2 > 0$), when there is an uncertainty whether σ_b^2 and / or σ_c^2 and/of σ_d^2 equal to zero. Under these situations (1) assumes the following forms:

(2) $y_{ijkmn} = \mu + a_i + b_{ij} + c_{ijk} + d_{ijkm} + e_{ijkmn}$, for $\sigma_b^2 > 0, \sigma_c^2 > 0, \sigma_d^2 > 0$

(3) $y_{ijkmn} = \mu + a_i + b_{ij} + c_{ijk} + e_{ijkmn}$, for $\sigma_b^2 > 0, \sigma_c^2 > 0, \sigma_d^2 > 0$

(4) $y_{ijkmn} = \mu + a_i + b_{ij} + d_{ijkm} + e_{ijkmn}$, for $\sigma_b^2 > 0, \sigma_c^2 > 0, \sigma_d^2 > 0$

(5) $y_{ijkmn} = \mu + a_i + b_{ij} + e_{ijkmn}$, for $\sigma_b^2 > 0, \sigma_c^2 => 0 = 0$

(6) $y_{ijkmn} = \mu + a_i + c_{ijk} + d_{ijkm} + e_{ijkmn}$, for $\sigma_b^2 = 0, \sigma_c^2 > 0, \sigma_d^2 > 0$

(7) $y_{ijkmn} = \mu + a_i + c_{ijk} + e_{ijkmn}$, for $\sigma_b^2 = 0, \sigma_c^2 > 0\ \sigma_d^2 = 0$

(8) $y_{ijkmn} = \mu + a_i + d_{ijkm} + e_{ijkmn}$, for $\sigma_b^2 = \sigma_c^2 = 0, \sigma_d^2 > 0$.

(9) $y_{ijkmn} = \mu + a_i + e_{ijkmn}$, for $\sigma_b^2 = \sigma_c^2 = \sigma_d^2 = 0$

Under the above cases (1) is called an incompletely speoified model. However, if it is known with certainty that $\sigma_b^2 > 0$, $\sigma_c^2 > 0$, $\sigma_d^2 > 0$, then the appropriate model is (2) and the model given by (1) is completely specified. On the other hand if it is know with certainty that $\sigma_b^2 = r_c^2 = \sigma_2^b = 0$, then the appropriate model is (9) and the model given by (1) is again completly speoified.

If we assume completely specified model given by (2), the analysis of variance will include all the components as given by Table I. The appropriate test for H. is to calculate the statistie $F_{10} = V_5/V_4$ and rejeet H_o if $F_{10} \geqq F(n_5, n_4 ; \alpha_{10})$ where $F(n_a, n_b ; \alpha)$ refers the upper 100α % point of F-distribution with n_a and n_b degree of freedom. This test is called the «Never Pool Test». If we assume the completely speoified model given by (9). then Table I will no longer include the components σ_b^2 , σ_c^2 and σ_d^2. In this case the appropriate test for H. is to calculate the statistic $F_{20} = V_5 (n_1 + n_2 + n_3 + n_4) / (n_1 V_1 + n_2 V_2 + n_3 V_3 + n_4 V_4)$ and reject H_o if $F_{20} \geq F (n_5 , n_1 + n_2 + n_3 + n_4 \alpha_{20})$ This test is called the «Always Pool Test». However, if we assume the incompletely speoified model, the proposed «Sometimes Pool Test Procedure» (SPTP) as a test for H_o consists in rejecting H_o if any one of the following mutually exclusive contingencies occur :

A_1: $V_4 / V_3 \geq F_1$ and $V_5 / V_4 \geq F_2$,

A_2: $V_4 / V_3 < F_1$, $V_{34} / V_2 \geq F_3$ and $V_5 / V_{34} \geq F_5$.

A_3: $V_4 / V_3 < F_1$, $V_{34} / V_2 < F_3$, $V_{234} / V_1 \geq F_4$ and $V_5 / V_{234} \geq F_6$;

A_4: $V_4 / V_3 < F_1$, $V_{34} / V_2 < F_3$, $V_{234} / V_1 < F_4$ and $V_5 / V_{1234} \geq F_7$;

Where

$V_{34} = (n_3 V_3 + n_4 V_4) / n_{34}$, $V_{234} = (n_2 V_2 + n_3 V_3 + n_4 V_4) / n_{234}$,

$V_{1234} = (n_1 V_1 + n_2 V_2 + n_3 V_3 + n_4 V_4) / n_{1234}$, $n_{ob} = n_a + n_b$.

$n_{abc} = n_{ab} + n_c$, $n_{abcd} = + n_{abc} + n_d$;

$F_1 = F(n_4 , n_3 ; \alpha_1)$, $F_2 = F(n_5 , n_4 , \alpha_2)$, $F_3 = F(n_{34} , n_2 ; \alpha_3)$,

$F_4 = F(n_{234} , n_1 ; \alpha_4)$, $F_5 = F(n_5 , n_{34} ; \alpha_5)$.

$F_6 = F(n_5 , n_{234} ; \alpha_6)$ and $F_7 = F(n_5 , n_{1234} ; \alpha_7)$

It may be remarked that the above nested classification in analysis of variance is not only the situation to which the proposed SPTP can be applied but also to the completely crossed and nested factorial experiments with four factors, all factors being random.

Let P (A_i) denote the probability of the event A_i (i = 1, 2, 3, 4). The probability P of rejecting H_o which is the power of SPTP is given by $P_1 + P_2 + P_3 + {}_4P$. This power P is function of 16 parameters, namely 5 degree of freedom n_1, n_2, n_3, n_4, n_5 ; 7 levels of significance α_1 , α_3 , α_4 (preliminary), σ_2 , α_5 , α_o , α_7 (final) and 4 raties of population variances (nuisance parameters) namely $\theta_{12} = \sigma_2^2 / \sigma_1^2$, $\theta_{32} = \sigma_3^2 / \sigma_2^2$, $\theta_{43} = \sigma_4^2 / \sigma_3^2$ and $\theta_{54} = \sigma_5^2 / \sigma_4^2$. In particular, when $\theta_{54} = 1.0$, the power P reduces to the size of the test procedure.

INTEGRAL EXPRESSIONS FOR POWER

The joint density of five independent mean squares can be written as.

$$(10)\quad g(V_1, V_2, V_3, V_4, V_5) = C\left[\prod_{i=1}^{5} V_i^{\frac{1}{2}n_i-1}\right]\exp\left[-\tfrac{1}{2}\sum_{i=1}^{5} n_i V_i/\sigma_i^2\right]$$

where the constant C is independent of V_i, s. Let us make the transformation

$$u_1 = \frac{n_2 V_2}{n_1 V_1 \theta_{21}},\quad u_2 = \frac{n_3 V_3}{n_2 V_2 \theta_{32}},\quad u_3 = \frac{n_4 V_4}{n_3 V_3 \theta_{43}},\quad u_4 = \frac{n_5 V_5}{n_4 V_4 \theta_{54}},\quad W = \frac{n_1 V_1}{n_5}$$

and integrate out w as gamma variate, we obtain the joint distribution of u_1, u_2, u_3, u_4 as

$$(11)\quad f(u_1, u_2, u_3, u_4) = K\,\frac{u_1^{a_{2345}-1}\, u_2^{a_{345}-1}\, u_3^{a_{45}-1}\, u_4^{a_5-1}}{(1+u_1+u_1u_2+u_1u_2u_3+u_1u_2u_3u_4)^{a_{12345}}},$$

where

$$K = \frac{\Gamma a_{12345}}{\Gamma a_1 \cdot \Gamma a_2 \cdot \Gamma a_3 \cdot \Gamma a_4 \cdot \Gamma a_5};\qquad a_i = \tfrac{1}{2} n_i \quad (i = 1, 2, \ldots, 5).$$

$$a_{45} = a_4 + a_5,\quad a_{345} = a_3 + a_{45},\quad a_{2345} = a_2 + a_{345},\quad a_{12345} = a_1 + a_{2345}$$

The four components of power P_1, P_2, P_3 and P can be written as where.

$$(12)\quad P_1 = \int_{u_3=a}^{\infty}\int_{u_4=b}^{\infty}\int_{u_1=0}^{\infty}\int_{u_2=0}^{\infty} f(u_1, u_2, u_3, u_4)\, du_3\, du_4\, du_1\, du_2 \,.$$

$$(13)\quad P_2 = \int_{u_3=0}^{a}\int_{u_2=\lambda_1}^{\infty}\int_{u_4=\lambda_2}^{\infty}\int_{u_1=0}^{\infty} f(u_1, u_2, u_3, u_4)\, du_3\, du_2\, du_4\, du_1 \,,$$

$$(14)\quad P_3 = \int_{u_3=0}^{a}\int_{u_2=0}^{\lambda_1}\int_{u_1=\lambda_3}^{\infty}\int_{u_4=\lambda_4}^{\infty} f(u_1, u_2, u_3, u_4)\, du_3\, du_2\, du_1\, du_4 \,,$$

and

$$(15)\quad P_4 = \int_{u_3=0}^{a}\int_{u_2=0}^{\lambda_1}\int_{u_1=0}^{\lambda_3}\int_{u_4=\lambda_5}^{\infty} f(u_1, u_2, u_3, u_4)\, du_3\, du_2\, du_1\, du_4$$

where

$$a = n_4 F_1 / n_3 \theta_{43}, \quad C = n_{34} F_3 / n_1 \theta_{32}, \quad d = n_{234} F_4 / n_1 \theta_{21},$$

$$b = n_3 F_2 / n_4 \theta_{33}, \quad e = n_5 F_7 / n_{1234} \theta_{54} \theta_{43} \theta_{32} \theta_{21},$$

$$f = n_5 F_6 / n_{234} \theta_{54} \theta_{43} \theta_{32}, \quad g = n_5 F_5 / n_{34} \theta_{54} \theta_{43}.$$

$$e_1 = e\theta_{21}, \quad e_2 = e_1 \theta_{32}, \quad e_3 = e_2 \theta_{43}, \quad f_1 = f\theta_{32}, \quad f_2 = f_1 \theta_{43},$$

$$g_1 = g\theta_{43}, \quad \lambda_1 = c/(1 + u_3\theta_{43}), \quad \lambda_2 = (g + g_1 u_3)/u_2,$$

$$\lambda_3 = d/(1 + u_2\theta_{32} + u_1 u_3 \theta_{32}\theta_{43}), \quad \lambda_4 = (f + f_1 u_4 + f_2 u_2 u_3)/u_2 u_3$$

$$\text{and } \lambda_5 = (e + e_1 u_1 + e_2 u_1 u_2 + e_3 u_1 u_2 u_3)/u_1 u_2 u_3$$

3.1 *Power Fourmulas* :

First we integrate P_1. From (11) and (12) we have

$$P_1 = k \int_a^\infty \int_b^\infty \int_0^\infty \int_0^\infty \frac{u_1^{a_{2345}-1} \; u_2^{a_{345}-1} \; u_3^{a_{45}-1} \; u_4^{a_5-1}}{(1 + u_1 + u_1 u_2 + u_1 u_2 u_3 + u_1 u_2 u_3 u_4)^{a_{12345}}} \, du_3 \, du_4 \, du_1 \, du_2$$

Integrating out u_1 and u_2 as beta variates of second kind, we obtain

$$P_1 = KB(a_{2345}, a_1)\, B(a_{345}, a_2) \int_a^\infty \int_b^\infty \frac{u_3^{a_{45}-1} \; u_4^{a_5-1}}{(1 + u_3 + u_3 u_4)^{a_{345}}} \, du_3 \, du_4 .$$

Let us make the transformation $x = (1 + u_3) / (1 + u_3 + u_3 u_u)$ and integrate out x, we obtain

$$P_1 = KB(a_{2345}, a_1)\, B(a_{345}, a_2)\, S_i \int_a^\infty \frac{u_3^{a_4-1} (1 + u_3)^i}{(1 + u_3 + bu_3)^{a_{34}+i}} \, du_3 ,$$

$$S_i = \sum_{i=0}^{a_5-1} (-1)^i \binom{a_5-1}{i}$$

Expanding $(1 + u_3)$ making the transformation $y = 1/(1 + u_3 + bu_3)$ and integrating out y, we obtain

$$P_1 = KB(a_{2345}, a_1)\, B(a_{345}, a_2)\, S_{ij}\, B_{x_1}(a_3 + i - j,\; a_4 + j)/(1+b)^{a_4+j}$$

where

$$S_{ij} = S_i \sum_{j=0}^{i} \binom{i}{j} \quad ; \quad x_1 = (1 + a + ab)^{-1} .$$

Proceeding as above we can obtain formulas for P_2, P_3. and P_4 Adding and simplifying we get the final expression for power P of SPTP as below :

$$P = S_{ij}\Big[\frac{k_1 B_{x_1}(a_3+i-j,\ a_4+j)}{(a_{34}+i)(1+b)^{a_4+j}} + \frac{k_2 S_{k'} S_\ell}{(a_{234}+i)}\Big\{\frac{B_{x_2}(a_4+\ell,\ a_3+i-\ell)}{(1+g)^{a_3+j-\ell}(1+g_1)^{a_4+\ell}} -$$

$$\frac{S_{m'}\, c^{a_{34}+j+k'}(1+g)^{k'-m'}(1+g_1)^{m'} B_{x_3}(a_4+\ell+m',\ a_3+j+k'-\ell-m')}{(\theta_{43}+c+cg_1)^{a_4+\ell+m'}(1+c+cg)^{a_3+j+k'-\ell'-m'}}\Big\} + \frac{k S_k S_\ell}{(a_{1234}+i)}$$

$$\Big\{\frac{S_{n'} S_p S_{q'}(1+f_1)^{n'-q'}(1+f_2)^{q'} B_{x_4}(a_4+p+q',\ a_3+\ell+n'-p-q')}{H_1^{a_3+\ell+n'-p-q'}\ H_2^{a_4+p+q'}\ (1+f)^{a_2+j-\ell}} + S_m S_n S_p S_q S_r$$

$$\Big\{\frac{(1+e_1)^{k-m}(1+e_2)^{m-q}(1+e_3)^{q} H_6^{n-r} H_7^{r} B_{x_5}(a_4+p+q+r,\ a_3+\ell+m+n-p-q-r)}{(1+e)^{a_1+i-j}\ H_5^{a_2+j+k-\ell-m}\ H_8^{a_3+\ell+m+n-p-q-r}\ H_9^{a_4+p+q+r}}$$

$$- \frac{(1+f)^{k-m}(1+f_1)^{m-q}(1+f_2)^{q}(\theta_{32}+d+df_1)^{n-r}(\theta_{43}\theta_{32}+d+df_2)^{r} B_{x_6}(a_4+p+q+r,\ a_3+\ell+m+n-p-q-r)}{(1+d+df)^{a_2+j+k-\ell-m}\ H_3^{a_3+\ell+m+n-p-q-r}\ H_4^{a_4+p+q+r}}\Big\}\Big\}\Big]$$

where

$$K_1 = \frac{\Gamma a_{345}}{\Gamma a_3 \cdot \Gamma a_4 \cdot \Gamma a_5}\ ,\qquad K_2 = \frac{\Gamma a_{2345}}{\Gamma a_2 \cdot \Gamma a_3 \cdot \Gamma a_4 \cdot \Gamma a_5}$$

$$x_2 = \frac{a(1+g_1)}{(1+g+a+ag_1)}\ ,\qquad x_3 = \frac{a(\theta_{43}+C+Cg_1)}{(1+C+Cg+a(\theta_{43}+C+Cg_1)}$$

$$x_4 = aH_2/(H_1+aH_2)\ ,\quad x_5 = aH_9/(aH_9+H_8)\ ,\quad x_6 = aH_4/(H_3+aH_4)$$

$$H_1 = 1+f+c+cf_1\ ,\quad H_2 = c+\theta_{43}+f\theta_{43}+cf_2\ .$$

$$H_3 = 1+d+df+c\theta_{32}+cd(1+f_1)\ ,\quad H_4 = \theta_{43}(1+d+df)+c\theta_{32}\theta_{43}+cd(1+f_1),$$

$$H_5 = 1+e+d(1+e_1)\ ,\quad H_6 = (1+e)\theta_{43}+d(1+e_2).$$

$$H_7 = (1+e)\theta_{32}\theta_{43}+d(1+e_3)\ ,\ H_8 = 1+e+d+de_1+c(1+e)\theta_{32}+cd(1+e_1),$$

$$H_9 = \theta_{43}(1+e+d+de_1)+c(1+e)\theta_{32}\theta_{43}+cd(1+e_3)\ ,$$

$$S_{k'} = \sum_{k'=0}^{a_2+i-j-1}(-1)^{k'}\frac{\binom{a_2+i-j-1}{k'}}{(a_{34}+j+k)}\qquad S_{m'} = \sum_{m'=0}^{k'}\binom{k'}{m'}$$

$$S_{n'} = \sum_{n'=0}^{a_2+j-\ell-1}(-1)^{n'}\frac{\binom{a_2+j-\ell-1}{n'} c^{a_{34}+\ell+n'}}{(a_{34}+\ell+n')}\ ,\quad S_{q'} = \sum_{q'=0}^{n'}\binom{n'}{q'}$$

$$S_k = \sum_{k=0}^{a_1+i-j-1}(-1)^k\frac{\binom{a_1+i-j-1}{k} d^{a_{234}+j+k}}{(a_{234}+j+k)}\ ,\quad S_\ell = \sum_{\ell=0}^{j}\binom{j}{\ell}$$

$$S_m = \sum_{m=0}^{k}\binom{k}{m}\ ,\quad S_n = \sum_{n=0}^{a_2+j+k-\ell-m-1}(-1)^n\frac{\binom{a_2+j+k-\ell-m-1}{n}}{(a_{34}+\ell+m+n)}\ ,$$

$$S_p = \sum_{p=0}^{\ell}\binom{\ell}{p}\ ,\quad S_q = \sum_{q=0}^{m}\binom{m}{q}\ ,\quad S_r = \sum_{r=0}^{n}\binom{n}{r}$$

DISCUSSION OF RESULTS

In this section we shall discuss the results of size and power of SPTP. The discussion is based on the theoretical results obtained in Section 3 and the numerical results assembled in Appendix II. As pointed out earlier, the power of SPTP is a function of 16 parameters. According to Bozivich, Bancroft and Hartely (2) the degrees of freedom, n_1 , n_2 n_3 , n_4 , $n_5$are completely deterined by the experiment; the nuisance parameters θ_{21}, θ_{32} , θ_{43} , θ_{54} are in general, unkown and hence none of these 9 parameters are at the disposal of the experimenter. The final levels of significance α_2, α_5 , α_6 , α_7 are chosen in advance and taken to be equal to .05 Hence only 3 pfeliminary levels of significance α_1 α_3 $\alpha_4$are at the choice of the experimenter. Here we have taken $\alpha_1=\alpha_3=\alpha_4=\alpha_i$and in this situation the power of SPTP is a function of 11, parameters only. The choice of the preliminary level of significance is made in such a way that the size of SPTP remains in the vicinity of the prescribed final level of significance and there is a gain in power of SPTP over the NPT. The study of Bozivich, Bancroft and Hartley (1) and Srivastava (6), (8) involving one and two doubtful error mean squares respectively indicated that for final levels of significance set at .05 the .05 preliminary level of significance often resulted in a size peak of less than .10. In order to control the size disturbances, we have, therefore, decided to study size and power of SPTP at. 25 preliminary level of significance for various combinations of degrees of freedom. The results of size of SPTP are also valid to the case of Mixed Models obtained from four-fold nested classificateion and four-fold nested and crossed classification assuming treatment effects as fixed effects and others random.

Jain and Gupta (4) showed that for $\theta_{43}= \theta_{32} = \theta_{21}=1.0$ the lower and upper bounds for size of SPTP are given by $(1 - \alpha_1)^3\alpha_2$ and $\alpha_2\,(2-\alpha_1)\,(2-2\;\alpha_1 + \alpha_1^2)$ respectively, where α_1 is preliminary level of significance and α_2 the final level of significance. These expressions are independent of the degrees of freedom. The lower bounds for priliminary level of significance. .01, 05., .25 are .0485, .0429, .0211 and the upper bounds are .1970, .1855, .1367 respedtively.

Tables 1 and 2 (Appendix II) give the size of SPTP for two sets of degrees of freedom with $\alpha_1 = .25$ and $\alpha_2 = .05$. We observe that the size of SPTP is a unimodel function of θ_{43} , θ_{32} , θ_{21} and approaches α_2 as θ's become large. It is due to the fact that as $\theta_{43} \rightarrow \infty$, (Appendix I), the probability of pooling approaches zero and SPTP approaches NPT. The size maximum of the test procedure varies both in magnitude and location. Table II gives the magnitude of size maximum for different sets of degree of freedom.

TABLE II

Magnitude of size maximum for $\theta_{43} = \theta_{32} = \theta_{21} = 1.0$

n^1	n^2	n^3	n^4	n^6	Size Maximum
2	2	2	2	2	.103
5	2	2	2	2	.104
10	2	2	2	2	.104
2	6	2	2	2	.106
2	10	2	2	2	.108
2	2	4	2	2	.113
2	2	10	2	2	.126
2	2	2	4	2	.075
2	2	2	8	2	.061
4	4	2	2	2	.104
8	4	2	2	2	.105
4	2	8	2	2	.122
4	2	16	2	2	.127
2	4	8	2	2	.122
4	4	4	2	2	.113
8	8	4	4	2	.075

We observe that for fixed prelminary levels of significance the size maximum

(*a*) Increases as n_1 incresses for fixed value of n_2 , n_3 , n_4 , n_5 ,

()) Increases as n_2 increases for fixed value of n_1 , n_3 , n_4 , n_5 ,

(*c*) Increases as n_3 increases for fixid value of n_1 , n_2 , n_4 , n_5 ,

(*d*) Decreases as n_4 increases for fixed values of n_1 , n_2 , n_3 , n_5 ,

(*e*) Increases as n_5 increases for fixed value of n_1 , n_2 , n_3 , n_4

Now if we fix the «reasonable tolerance» for size maximum at . 10, them most of the combimations of degrees of freedom at 28% prelininary level of significance will give adequate size control. Table Ivigives some of the satis factory combinatioins of degrees of freedom with upper limit to their sizs-maximum.

TABLE III

Satisfactory combination of degrees of freedom for 25% preliminary level of significance

n_1	n_2	n_3	n_4	n_5	Upper Limit to Size Maximum
≥ 2	2	2	2	2	.103
≤ 10	2	2	2	2	.104
≤ 8	4	2	2	2	.104
≥ 8	6	4	4	2	.075

Now we attempt a comparison of power of SPTP with that of NPT. As we donot have overall size of SPTP a constant, we have adopted the following method for power comparison.

(1) For given values of θ_{43} , θ_{32} , θ_{21} , compute the size of SPTP.

(11) For the size calculated in (i), compute the power of the two test procedures for specified values of θ_{54}. Tables 3 and 4 summarize the results of power gain of SPTP over the corresponding Never Pool Test for $\alpha_1 =. 25$ and $\alpha_2 = 05$ It is observed tsat for fixed values of θ_{43} , θ_{32} , θ_{21} and a given set of degrees of freedom, the power of SPTP and NPT is a monotone increasing function of θ_{54} . For $\theta_{43} = \theta_{32} = \theta_{21} = 1.\ 0$, SPTP is more powerful than the corresponding NPT. It is also observed that SPTP is more powerful for $\theta_{43} < 3.\ 0$, $\theta_{32} \leqslant 5.\ 0$, $\theta_{21} \leqslant 5.\ 0$ and is less powerful for $\theta_{43} \geqslant 3.0$, $\theta_{12} \geqslant 1.0$, $\theta_{21} \geqslant 1.0$, than the corresponding NPT for most of the combinations. When θ_{43} becomes large (tending to ∞), SPTP approaches the Never Pool Test and hence the power gain or loss approaches zero. From the tables it is observed that for fixed values of other parameters the magnitude of power gain or loss of SPTP increases as (i) n_1 or n_2 or n_3 increases, (ii) n_4 decreases, (iii) n_5 increases.

On the basis of the size and power results whish have been discussed, above, we now attempt recommendations regarding the use of SPTP studied. If the experisenter thinks that θ's are small and he uses the never pool test, then he shall be using a less power ful test. Similarly if θ's are not small and the experinenter usse the always pool test, then he shall again be using a less powerful test. If the experinenter has no idea about the magnitudes of θ's, then the use of SPTP incorporating some preliminary tests of sinificance is suggested. If SPTP is used, the prelininary levels of significance should be chosen with care so as to have as adequate size control. For the situations of the type described in Table IV, the 28% preliminary level of significance can be used.

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APPENDIX I

Reaults in terms of Size of SPTP :

Here we shall prove some mathematical results regarding the size of SPTP.

Result 1. As θ_{43} becomes large (approaching infinity), SPTP approaches NPT and the size of SPTP approaches the final level of significance α_2.

Proof: As $\theta_{43} \to \infty$ the power components P (A_2), P (A_3) and P (A_4) of SPTP tend to zéro and the component $P(A_1)$ approaches to

$$P(A_1) = k \int\int\int\int_{0}^{\infty} \frac{u_1^{a_{2345}-1}\; u_2^{a_{345}-1}\; u_3^{a_{45}-1}\; u_4^{a_5-1}}{(1+u_1+u_1u_2+u_1u_2u_3+u_1u_2u_3u_4)}\, du_1\, du_2\, du_3\, du_4$$

Integrating u_1 , u_2 and u_3 as beta variate of second kind, we obtain

$$P(A_1) = \frac{1}{B(a_5, a_4)} \int_{b}^{\infty} \frac{u_4^{a_5-1}}{(1+u_4)^{a_{45}}}\, du_4$$

This expression also represent the power of the never pool test. The size of SPTP is obtained by substituting $\theta_{54} = 1.0$ in (1), giving.

$$S(A_1) = \frac{1}{B(a_5, a_4)} \int_{u_2^0}^{\infty} \frac{u_4^{a_5-1}}{(1+u_4)^{a_{45}}}\, du_4 ,$$

where S (A_1) is the size of the never pool test and

$$u_2 = n_5 P_2/n_4.$$

Using the relationship between the F-distribution and incomplete beta function, we have

$$S\ A_1 = \alpha_2$$

Result 2. As θ_{32} becomes large (approaching infinity) and for $\theta_{43} = 1.0$, the size of SPTP is less than α_2 $(2-\alpha_1)$, where α_1 and α_2 are preliminary and final levels of significance respectively.

Result 3. As θ_{21} becomes large (approaching infinity and for $\theta_{43} = \theta_{32} = 1.0$, the size of SPTP is less than $\alpha_2 [3(1-\alpha_1) + \alpha_1^2)$, where α_1 and α_2 are preliminary and final levels of significances respectively.

The Results 2 and 3 can be proved on the lines of Result. 1.

APPENDIX II

TABLE 1

Size of SPTP for $n_1 = n_2 = n_3 = 4, n_4 = n_5 = 2.$
$\alpha_1 = \alpha_3 = \alpha_4 = .25, \alpha_1 = \sigma_5 = \alpha_7 = \alpha_6 = .05$

		θ_{21}		
θ_{43}	θ_{42}	1.0	3.0	5.0
1.0	1.0	.0263	.0341	.3330
	2.0	.0445	.0463	.0447
	3.0	.0489	.0483	.0473
	5.0	.0474	.0462	.0458
5.0	1.0	.1093	.1110	.1097
	2.0	.1127	.1119	.1114
	3.0	.1096	.1089	.1087
10.0	1.0	.1016	.1019	.1014
	2.0	.1015	.1011	.1009
	3.0	.0997	.0995	.0995

TABLE 2

Size of SPTP for $n_1 = 8, n_2 = 6, n_3 = n_4 = 4, n_5 = 2$
$\alpha_1 = \alpha_3 = \alpha_4 = .25, \alpha_2 = \alpha_5 = \alpha_6 = \alpha_7 = \cdot 05$

		θ_{21}		
θ_{43}	θ_{32}	1.0	2.0	3.0
1.0	1.0	.0276	.0345	.0342
	3.0	.0475	.0470	.0463
	5.0	.0440	.0435	.0434
5.0	1.0	.0751	.0753	.0748
	3.0	.0719	.0718	.0717
	5.0	.0708	.0707	.0707
10.0	1.0	.0633	.0633	.0732
	3.0	.0619	.0618	.0618
	5.0	.0616	.0616	.0616

TABLE 3

Power Gain of PSTP over the Never Pool Test of the Same Size
for $n_1 = 4, n_2 = 4, n_3 = 4, n_4 = n_5 = 2.$
$\alpha_1 = \alpha_3 = \alpha_4 = .25, \quad \alpha_2 = \alpha_5 = \alpha_6 = \alpha_7 = .05$

θ_{43}	θ_{32}	θ_{21}	1.0	5.0	10.0	50.0
1.0	1.0	1.0	0	.2009	.2839	.2344
		3.0	.0	.1780	.2412	.1719
		5.0	0	.1793	.2449	.1790
	3.0	1.0	0	.1325	.1644	.0898
		3.0	0	.1327	.1660	.0921
		5.0	0	.1355	.1705	.0965
3.0	1.0	1.0	0	—.0077	—.0578	—.0774
		3.0	0	—.0174	—.0691	—.0837
		5.0	0	—.0133	—.0641	—.0808
	3.0	1.0	0	—.0287	—.0802	—.0888
		3.0	0	—.0264	—.0774	—.0872
		5.0	0	—.0257	—.0766	—.0869
5.0	1.0	1.0	0	—.0855	—.1425	—.1168
		3.0	0	—.0871	—.1439	—.1175
		5.0	0	—.0867	—.1436	—.1173
	3.0	1.0	0	—.0886	—.1448	—.1175
		3.0	0	—.0871	—.1441	—.1167
		5.0	0	—.0866	—.1426	—.1164

TABLE 4

Power Gain of SPTP over the Never Pool Test of the Same Size
for $n_1 = 8$, $n_2 = 6$, $n_3 = n_4 = 4$, $n_5 = 2$
$\alpha_1 = \alpha_3 = \alpha_4 = \alpha_5 = .25$, $\alpha_3 = \alpha_5 = \alpha_6 = \alpha_7 = .05$

θ_{43}	θ_{32}	θ_{21}	1 .0	§ .0	10 .0	50 .0
1 .0	1 .0	1 .0	0	.1285	.1452	.0638
		3 .0	0	.1016	.1204	.0560
		5 .0	0	.1068	.1172	.0497
	3 .0	1 .0	0	.0528	.0543	.0202
		3 .0	0	.0560	.0572	.0219
		5 .0	0	.0671	.0584	.0224
	5 .0	1 .0	0	.0582	.0616	.0246
		3 .0	0	.0606	.0641	.0254
		5 .0	0	.0615	.0651	.0260
3 .0	1 .0	1 .0	0	—.0280	—.0368	—.0172
		3 .0	0	—.0299	—.0384	—.0249
		5 .0	0	—.0515	—.0378	—.0184
	3 .0	1 .0	0	—.0295	—.0402	—.0186
		3 .0	0	—.0284	—.0361	—.0166
		5 .0	0	—.0284	—.0361	—.0166
5 .0	1 .0	1 .0	0	—.0517	—.0562	—.0231
		3 .0	0	—.0512	—.0557	—.0229
		5 .0	0	—.0504	—.0549	—.0226
	3 .0	1 .0	0	—.0453	—.0496	—.0206
		3 .0	0	—.0447	—.0492	—.0204
		5 .0	0	—.0447	—.0492	—.0204
	5 .0	1 .0	0	—.0426	—.0471	—.0196
		3 .0	0	—.0422	—.0469	—.0194
		.0	0	—.0422	—.0469	—.0194

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