

**FURTHER EXPRESSIONS OF TRIVARIATE K-STATISTICS
IN TERMS OF POWER SUMS
AND SAMPLE MOMENTS OF ORDER 7**

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§ 1.1 The k-statistics are of interest to workers in the theory of sampling distributions and moment statistics. They are the basis for deriving the tables of symmetric function «See David and others . . (1966), Mikhail (1968) and ZIA-UD-DIN (1954, 55, 59).

Recently the properties of k-statistics have been applied in combinatory analysis analysis of variance, design of experiments, genetics and industrial experiments.

§ 1.2 We used Kendall Operator (1940) in deriving the expressions of the trivariate k-statistics. We checked the expressions using Dressel (1940) method. We limited this paper to include the expressions of K-statistics of order 7 only. Now work is on expressions of higher orders and also on constructing tables of trivariate symmetric function of these higher orders.

§ 1.3 The remarkable notice is that these trivariate expressions of order ($w=7$) give the number of partitions of tripartite number ($a+b+c=w$) which include those of unipartitions and bipartitions given by Gupta (1951) and by David-Kendall-Barton (1966) respectively.

The tables of tripartitions « See Mikhail (1966) and Mikhail and Attalla (1968)» made a double check to the given expressions, in table 1.

The k-statistics of order 7

$$\begin{aligned}
 k_{700} = n^{-(7)} & (n^6 + 42 n^5 + 119 n^4 - 42 n^3) S_{700} - 7(n^5 + 42 n^4 + 119 n^3 \\
 & - 42 n^2) S_{600} S_{100} - 21(n^5 + 12 n^4 - 31 n^3 + 18 n^2) S_{500} S_{200} \\
 & - 35(n^5 + 5 n^3 - 6 n^2) S_{400} S_{300} + 42(n^4 + 27 n^3 + 44 n^2 - 12 n) \\
 & S_{500} S_{100}^2 + 210(n^4 + 6 n^3 - 13 n^2 + 6 n) S_{400} S_{200} S_{100} + 140(n^4 + \\
 & 5 n^2 - 6 n) S_{300} S_{100}^2 + 210(n^4 - 3 n^3 + 2 n^2) S_{300} S_{200}^2 - 210(n^3 \\
 & + 13 n^2 + 6 n) S_{400} S_{100}^3 - 1260(n^3 + n^2 - 2 n) S_{300} S_{200} S_{100}^2 - 630 \\
 & (n^3 - 3 n^2 + 2 n) S_{200} S_{100}^3 + 840(n^2 + 4 n) S_{300} S_{100}^4 + 2520(n^2 - \\
 & n) S_{200} S_{100}^2 S_{100}^3 - 2520(n) S_{200} S_{100}^5 + 720 S_{100}^7
 \end{aligned}$$

This can be written in the following form :

$$\begin{aligned}
 k_{700} = a_0 \{ & a_1 S_{700} - 7 a_2 S_{600} S_{100} - 21 a_3 S_{500} S_{200} - 35 a_4 S_{400} S_{300} + 42 a_5 \\
 & S_{500} S_{100}^2 + 210 a_6 S_{400} S_{200} S_{100} + 140 a_7 S_{300} S_{100}^2 + 210 a_8 S_{300} S_{200}^2 \\
 & - 210 a_9 S_{400} S_{100}^3 - 1260 a_{10} S_{300} S_{200} S_{100}^2 - 630 a_{11} S_{200} S_{100}^3 + 840 \\
 & a_{12} S_{300} S_{100}^4 + 2520 a_{13} S_{200} S_{100}^2 S_{100}^3 - 2520 a_{14} S_{200} S_{100}^5 + 720 S_{100}^7 \} \\
 k_{610} = a_0 \{ & a_1 S_{610} - 6 a_2 S_{510} S_{100} - a_3 S_{600} S_{010} - 15 a_4 S_{410} S_{200} - 6 a_5 S_{350} S_{110} \\
 & - 20 a_6 S_{310} S_{300} - 15 a_7 S_{440} S_{210} + 30 a_8 S_{410} S_{100}^2 + 12 a_9 S_{550} S_{100} S_{010} \\
 & + 120 a_{10} S_{6310} S_{200} S_{100} + 60 a_{11} S_{6400} S_{110} S_{100} + 30 a_{12} S_{6400} S_{200} S_{010} + 120 a_{13} \\
 & S_{300} S_{210} S_{100} + 20 a_{14} S_{7300} S_{010}^2 + 90 a_{15} S_{8210} S_{200}^2 + 120 a_{16} S_{8300} S_{200} S_{110} \\
 & - 120 a_{17} S_{9310} S_{100}^3 - 90 a_{18} S_{9400} S_{100} S_{010}^2 - 540 a_{19} S_{10210} S_{200} S_{100}^2
 \end{aligned}$$

$$\begin{aligned}
 & - 360 a_{10} s_{300} s_{110} s_{100}^2 - 360 a_{10} s_{300} s_{200} s_{100} s_{010} - 540 a_{11} s_{200}^2 \\
 & s_{110} s_{100} - 90 a_{11} s_{200}^3 s_{010} + 360 a_{12} s_{210} s_{100} s_{010}^4 + 480 a_{12} s_{300} s_{100} s_{010}^3 s_{010} \\
 & + 1440 a_{13} s_{200} s_{110} s_{100} s_{010}^3 + 1080 a_{13} s_{200} s_{100} s_{010}^2 s_{010}^2 s_{010} - 720 a_{14} s_{110} s_{100} s_{010}^5 - \\
 & 1800 a_{14} s_{200} s_{100} s_{010}^4 s_{010} + 720 s_{100}^6 s_{010} \}
 \end{aligned}$$

$$\begin{aligned}
 k_{520} = a_0 \{ & a_{1520} s_{110} - 5 a_{2420} s_{3500} s_{020} - 2 a_{2510} s_{4220} s_{300} - 10 a_{3320} s_{320} s_{200} - 10 a_{3410} s_{3410} \\
 & s_{110} s_{3500} s_{020} - 10 a_{4220} s_{4220} s_{300} - 20 a_{4310} s_{4310} s_{210} - 5 a_{4400} s_{4400} s_{120} + \\
 & 20 a_{5320} s_{5320} s_{100}^2 + 20 a_{5410} s_{5410} s_{100} s_{010} + 2 a_{5500} s_{5500} s_{010}^2 + 60 a_{6220} s_{6220} s_{200} s_{100} \\
 & + 80 a_{6310} s_{6310} s_{110} s_{100} + 40 a_{6310} s_{6310} s_{200} s_{010} + 10 a_{6400} s_{6400} s_{020} s_{100} + \\
 & 20 a_{6400} s_{6400} s_{110} s_{010} + 60 a_{7210} s_{7210} s_{100}^2 s_{010} + 40 a_{7300} s_{7300} s_{120} s_{100} + 40 a_{7300} s_{7300} s_{300} \\
 & s_{210} s_{010} + 30 a_{8120} s_{8120} s_{200} s_{010}^2 + 120 a_{8210} s_{8210} s_{200} s_{110} s_{010} + 40 a_{8300} s_{8300} s_{110} s_{010}^2 + \\
 & 20 a_{8300} s_{8300} s_{200} s_{020} - 60 a_{9220} s_{9220} s_{100} s_{010}^3 - 120 a_{9310} s_{9310} s_{100} s_{010}^2 s_{010} - 30 a_{9310} s_{9310} s_{100} s_{010}^2 \\
 & s_{400} s_{100} s_{010}^2 - 180 a_{10120} s_{10120} s_{200} s_{100} s_{010}^2 - 360 a_{10210} s_{10210} s_{110} s_{100} s_{010}^2 - 360 a_{10210} s_{10210} s_{110} s_{100} s_{010}^2 \\
 & s_{210} s_{200} s_{100} s_{010} - 60 a_{10300} s_{10300} s_{020} s_{100} s_{010}^2 - 240 a_{10300} s_{10300} s_{110} s_{100} s_{010}^2 s_{010} \\
 & - 60 a_{10300} s_{10300} s_{200} s_{010} s_{010}^2 - 360 a_{11200} s_{11200} s_{110} s_{100} s_{010}^2 - 90 a_{11200} s_{11200} s_{020} s_{100} s_{010}^2 \\
 & - 180 a_{11200} s_{11200} s_{110} s_{100} s_{010}^2 s_{010} + 120 a_{12120} s_{12120} s_{100} s_{010}^4 + 480 a_{12210} s_{12210} s_{100} s_{010}^3 s_{010} + \\
 & 240 a_{12300} s_{12300} s_{100} s_{010}^2 s_{010}^2 + 480 a_{13110} s_{13110} s_{100} s_{010}^2 s_{010}^3 + 240 a_{13200} s_{13200} s_{020} s_{100} s_{010}^3 + \\
 & 1440 a_{13200} s_{13200} s_{110} s_{100} s_{010}^2 s_{010} + 360 a_{13200} s_{13200} s_{100} s_{010}^2 s_{010}^2 - 120 a_{14020} s_{14020} s_{100} s_{010}^5 \\
 & - 1200 a_{14110} s_{14110} s_{100} s_{010}^4 s_{010} - 1200 a_{14200} s_{14200} s_{100} s_{010}^3 s_{010}^2 + 720 s_{100}^5 s_{010}^2 \}
 \end{aligned}$$

$$\begin{aligned}
 k_{511} = a_0 \{ & a S - 5 a S S - a S S - a S S - 10 a S S - \\
 & 1 \ 511 \quad 2 \ 411 \ 100 \quad 2 \ 501 \ 010 \quad 2 \ 510 \ 001 \quad 3 \ 311 \ 200 \\
 & 5 a S S - 5 a S S - a S S - 10 a S S - 10 a S S \\
 & 3 \ 401 \ 110 \quad 3 \ 410 \ 101 \quad 3 \ 500 \ 011 \quad 4 \ 211 \ 011 \quad 4 \ 211 \ 300 \\
 & - 10 a S S - 10 a S S - 5 a S S + 20 a S S^2 + \\
 & 4 \ 301 \ 210 \quad 4 \ 310 \ 201 \quad 4 \ 400 \ 111 \quad 5 \ 311 \ 100 \\
 & 10 a S S S + 10 a S S S + 2 a S S S + 60 a S S \\
 & 5 \ 401 \ 100 \ 010 \quad 5 \ 410 \ 100 \ 001 \quad 5 \ 500 \ 010 \ 001 \quad 6 \ 211 \ 200 \\
 & S \\
 & 100 + 40 a S S S + 40 a S S S + 20 a S S S + \\
 & 6 \ 301 \ 110 \ 100 \quad 6 \ 310 \ 101 \ 100 \quad 6 \ 301 \ 200 \ 010 \\
 & 20 a S S S + 10 a S S S + 10 a S S S + 10 a S \\
 & 6 \ 310 \ 200 \ 001 \quad 6 \ 400 \ 011 \ 100 \quad 6 \ 400 \ 101 \ 010 \quad 6 \ 400 \\
 & S S + 60 a S S S + 40 a S S S + 20 a S S S + \\
 & 110 \ 001 \quad 7 \ 210 \ 201 \ 100 \quad 7 \ 300 \ 111 \ 100 \quad 7 \ 300 \ 201 \ 010 \\
 & 20 a S S S + 30 a S S^2 + 60 a S S S + 60 a S S \\
 & 7 \ 300 \ 210 \ 001 \quad 8 \ 111 \ 200 \quad 8 \ 201 \ 200 \ 110 \quad 8 \ 210 \ 200 \\
 & S + 40 a S S S + 20 a S S S - 60 a S S^3 - 60 a S \\
 & 101 \quad 8 \ 300 \ 110 \ 101 \quad 8 \ 300 \ 200 \ 011 \quad 9 \ 211 \ 100 \quad 9 \ 301 \\
 & S^2 S - 60 a S S^2 S - 30 a S S S S - 180 a S S \\
 & 100 \ 010 \quad 9 \ 310 \ 100 \ 001 \quad 9 \ 400 \ 100 \ 010 \ 001 \quad 10 \ 111 \ 200 \\
 & S^2 - 180 a S S S^2 - 180 a S S S^2 - 60 a S S S^2 - \\
 & 100 - 180 \ 10 \ 201 \ 110 \ 100 \quad 10 \ 210 \ 101 \ 100 \quad 10 \ 300 \ 011 \ 100 \\
 & - 120 a S S S S - 120 a S S S S - 180 a S S \\
 & 10 \ 300 \ 101 \ 100 \ 010 \quad 10 \ 300 \ 110 \ 100 \ 001 \quad 10 \ 201 \ 200 \\
 & S S - 180 a S S S S - 60 a S S S S - 360 a \\
 & 100 \ 010 - 180 \ 10 \ 210 \ 200 \ 100 \ 001 \quad 10 \ 300 \ 200 \ 010 \ 001 \quad 11 \\
 & S S S S - 90 a S^2 S S - 90 a S^2 S S - 90 a S^2 \\
 & 200 \ 110 \ 101 \ 100 \quad 11 \ 200 \ 011 \ 100 \quad 11 \ 200 \ 101 \ 010 \quad 11 \ 200 \\
 & S S + 120 a S S^4 + 240 a S S^3 S + 240 a S S^3 S \\
 & 110 \ 001 \quad 12 \ 111 \ 100 \quad 12 \ 201 \ 100 \ 010 \quad 12 \ 210 \ 100 \ 001 \\
 & + 240 a S S^2 S S + 480 a S S S^3 + 240 a S S S^3 \\
 & 12 \ 300 \ 100 \ 010 \ 001 \quad 13 \ 110 \ 101 \ 100 \quad 13 \ 200 \ 011 \ 100 \\
 & + 720 a S S S^2 S + 720 a S S S^2 S + 360 a S^2 S \\
 & 13 \ 200 \ 101 \ 100 \ 010 \quad 13 \ 200 \ 110 \ 100 \ 001 \quad 13 \ 200 \ 100 \\
 & S S - 120 a S S^5 - 600 a S S^4 S - 600 a S S^4 S \\
 & 010 \ 001 \quad 14 \ 011 \ 100 \quad 14 \ 101 \ 100 \ 010 \quad 14 \ 110 \ 100 \ 001 \\
 & - 1200 a S S^3 S S + 720 S^5 S S \} \\
 & 14 \ 200 \ 100 \ 010 \ 001 \quad 100 \ 010 \ 001 \}
 \end{aligned}$$

$$\begin{aligned}
 x_{430} = a_0 \{ & \frac{a S}{1 \ 430} - \frac{4 a S}{2 \ 330} \frac{S}{100} - \frac{3 a S}{2 \ 420} \frac{S}{010} - \frac{6 a S}{3 \ 230} \frac{S}{200} - \frac{12 a S}{3 \ 320} \frac{S}{110} \\
 & - \frac{3 a S}{3 \ 410} \frac{S}{020} - \frac{4 a S}{4 \ 130} \frac{S}{300} - \frac{18 a S}{4 \ 220} \frac{S}{210} - \frac{12 a S}{4 \ 310} \frac{S}{120} + \frac{a S}{4 \ 400} \frac{S}{030} \\
 & + \frac{12 a S}{5 \ 230} \frac{S^2}{100} + \frac{24 a S}{5 \ 320} \frac{S}{100} \frac{S}{010} + \frac{6 a S}{5 \ 410} \frac{S^2}{010} + \frac{24 a S}{6 \ 130} \frac{S}{200} \frac{S}{100} \\
 & + \frac{72 a S}{6 \ 220} \frac{S}{110} \frac{S}{100} + \frac{36 a S}{6 \ 220} \frac{S}{200} \frac{S}{010} + \frac{24 a S}{6 \ 310} \frac{S}{020} \frac{S}{100} + \frac{48 a S}{6 \ 310} \\
 & \frac{S}{110} \frac{S}{010} + \frac{6 a S}{6 \ 400} \frac{S}{020} \frac{S}{010} + \frac{72 a S}{7 \ 210} \frac{S}{120} \frac{S}{100} + \frac{36 a S^2}{7 \ 210} \frac{S}{010} + \frac{8 a S}{7 \ 300} \\
 & \frac{S}{030} \frac{S}{100} + \frac{24 a S}{7 \ 300} \frac{S}{120} \frac{S}{010} + \frac{6 a S}{8 \ 030} \frac{S^2}{200} + \frac{72 a S}{8 \ 120} \frac{S}{200} \frac{S}{110} + \frac{72 a S}{8 \ 210} \\
 & \frac{S^2}{110} + \frac{36 a S}{8 \ 210} \frac{S}{200} \frac{S}{020} + \frac{24 a S}{8 \ 300} \frac{S}{110} \frac{S}{020} - \frac{24 a S}{9 \ 130} \frac{S^3}{100} - \frac{108 a S}{9 \ 220} \\
 & \frac{S^2}{100} \frac{S}{010} - \frac{72 a S}{9 \ 310} \frac{S}{100} \frac{S}{010} - \frac{6 a S}{9 \ 400} \frac{S^3}{010} - \frac{36 a S}{10 \ 030} \frac{S}{200} \frac{S^2}{100} - \frac{216 a}{10} \\
 & \frac{S}{120} \frac{S}{110} \frac{S^2}{100} - \frac{216 a}{10} \frac{S}{120} \frac{S}{200} \frac{S}{100} \frac{S}{010} - \frac{108 a}{10} \frac{S}{210} \frac{S}{020} \frac{S^2}{100} - \frac{432 a}{10} \\
 & \frac{S}{210} \frac{S}{110} \frac{S}{100} \frac{S}{010} - \frac{108 a}{10} \frac{S}{210} \frac{S}{200} \frac{S^2}{010} - \frac{72 a}{10} \frac{S}{300} \frac{S}{020} \frac{S}{100} \frac{S}{010} - \frac{72 a}{10} \\
 & \frac{S}{300} \frac{S}{110} \frac{S^2}{010} - \frac{144 a}{11} \frac{S^3}{110} \frac{S}{100} - \frac{216 a}{11} \frac{S}{200} \frac{S}{110} \frac{S}{020} \frac{S}{100} - \frac{216 a}{11} \frac{S}{200} \\
 & \frac{S^2}{110} \frac{S}{010} - \frac{54 a}{11} \frac{S^2}{200} \frac{S}{020} \frac{S}{010} + \frac{24 a}{12} \frac{S}{030} \frac{S^4}{100} + \frac{288 a}{12} \frac{S}{120} \frac{S}{100} \frac{S^3}{010} \\
 & + \frac{432 a}{12} \frac{S}{210} \frac{S^2}{100} \frac{S^2}{010} + \frac{96 a}{12} \frac{S}{300} \frac{S}{100} \frac{S^3}{010} + \frac{288 a}{13} \frac{S}{110} \frac{S}{020} \frac{S^3}{100} \\
 & + \frac{864 a}{13} \frac{S}{110} \frac{S^2}{100} \frac{S^2}{010} + \frac{432 a}{13} \frac{S}{200} \frac{S}{020} \frac{S^2}{100} \frac{S}{010} + \frac{864 a}{13} \frac{S}{200} \frac{S}{110} \frac{S}{100} \\
 & \frac{S^2}{010} + \frac{72 a}{13} \frac{S^2}{200} \frac{S^3}{010} - \frac{360 a}{14} \frac{S}{020} \frac{S^4}{100} \frac{S}{010} - \frac{1440 a}{14} \frac{S}{110} \frac{S^3}{100} \frac{S^2}{010} \\
 & - \frac{720 a}{14} \frac{S}{200} \frac{S^2}{100} \frac{S^3}{010} + \frac{720 a}{100} \frac{S^4}{010} \frac{S^3}{010} \}
 \end{aligned}$$

$$\begin{aligned}
 x_{421}^k = a_0 \{ & \frac{a S}{1 \ 421} - \frac{4 a S}{2 \ 321} \frac{S}{100} - \frac{2 a S}{2 \ 411} \frac{S}{010} - \frac{a S}{2 \ 420} \frac{S}{001} - \frac{6 a S}{3 \ 221} \frac{S}{200} \\
 & - \frac{8 a S}{3 \ 311} \frac{S}{110} - \frac{4 a S}{3 \ 320} \frac{S}{101} - \frac{a S}{3 \ 401} \frac{S}{020} - \frac{2 a S}{3 \ 410} \frac{S}{011} - \frac{4 a S}{4 \ 121} \frac{S}{300} \\
 & - \frac{12 a S}{4 \ 211} \frac{S}{210} - \frac{6 a S}{4 \ 220} \frac{S}{201} - \frac{8 a S}{4 \ 301} \frac{S}{120} - \frac{4 a S}{4 \ 310} \frac{S}{111} - \frac{a S}{4 \ 400} \frac{S}{021}
 \end{aligned}$$

$$\begin{aligned}
 & + 12 a S S^2 + 16 a S S S + 8 a S S S + 2 a S S^2 \\
 & \quad 5 \ 221 \ 100 \quad 5 \ 311 \ 100 \ 010 \quad 5 \ 320 \ 100 \ 001 \quad 5 \ 401 \ 010 \\
 & + 4 a S S S + 24 a S S S + 48 a S S S + 24 a S \\
 & \quad 5 \ 410 \ 010 \ 001 \quad 6 \ 121 \ 200 \ 100 \quad 6 \ 211 \ 110 \ 100 \quad 6 \ 220 \\
 & S S + 24 a S S S + 12 a S S S + 16 a S S S \\
 & \quad 101 \ 100 \quad 6 \ 211 \ 200 \ 010 \quad 6 \ 220 \ 200 \ 001 \quad 6 \ 301 \ 020 \ 100 \\
 & + 8 a S S S + 16 a S S S + 16 a S S S + 16 a S \\
 & \quad 6 \ 310 \ 011 \ 100 \quad 6 \ 301 \ 110 \ 010 \quad 6 \ 310 \ 101 \ 010 \quad 6 \ 311 \\
 & S S + 4 a S S S + 2 a S S S + 24 a S S S \\
 & \quad 110 \ 001 \quad 6 \ 400 \ 011 \ 010 \quad 6 \ 400 \ 020 \ 001 \quad 7 \ 201 \ 120 \ 100 \\
 & + 48 a S S S + 24 a S S S + 12 a S^2 S + 8 a S S \\
 & \quad 7 \ 210 \ 111 \ 100 \quad 7 \ 210 \ 201 \ 010 \quad 7 \ 210 \ 001 \quad 7 \ 300 \ 021 \\
 & S + 16 a S S S + 8 a S S S + 6 a S S^2 + 48 a \\
 & \quad 100 \quad 7 \ 300 \ 111 \ 010 \quad 7 \ 300 \ 120 \ 001 \quad 8 \ 021 \ 200 \quad 8 \\
 & S S S + 24 a S S S + 24 a S S^2 + 48 a S S S \\
 & \quad 111 \ 200 \ 110 \quad 8 \ 120 \ 200 \ 101 \quad 8 \ 201 \ 110 \quad 8 \ 210 \ 110 \ 101 \\
 & + 12 a S S S + 24 a S S S + 8 a S S S + 16 a S \\
 & \quad 8 \ 201 \ 200 \ 020 \quad 8 \ 210 \ 200 \ 011 \quad 8 \ 300 \ 101 \ 020 \quad 8 \ 300 \\
 & S S - 24 a S S^3 - 72 a S S^2 S - 36 a S S^2 S - 24 a \\
 & \quad 110 \ 011 \quad 9 \ 121 \ 100 \quad 9 \ 211 \ 100 \ 010 \quad 9 \ 220 \ 100 \ 001 \quad 9 \\
 & S S S^2 - 48 a S S S S - 6 a S S^2 S - 36 a S S \\
 & \quad 301 \ 100 \ 010 \quad 9 \ 310 \ 100 \ 010 \ 001 \quad 9 \ 400 \ 010 \ 001 \quad 10 \ 021 \ 200 \\
 & S^2 - 144 a S S S^2 - 72 a S S S^2 - 144 a S S S \\
 & \quad 100 \quad 10 \ 111 \ 110 \ 100 \quad 10 \ 120 \ 101 \ 100 \quad 10 \ 111 \ 200 \ 100 \\
 & S - 72 a S S S S - 42 a S S S^2 - 84 a S S S^2 \\
 & \quad 010 \quad 10 \ 120 \ 200 \ 100 \ 001 \quad 10 \ 201 \ 020 \ 100 \quad 10 \ 210 \ 011 \ 100 \\
 & - 138 a S S S S - 138 a S S S S - 138 a S S \\
 & \quad 10 \ 201 \ 110 \ 100 \ 010 \quad 10 \ 210 \ 110 \ 100 \ 010 \quad 10 \ 210 \ 110 \\
 & S S - 36 a S S S^2 - 72 a S S S S - 52 a S S \\
 & \quad 100 \ 001 \quad 10 \ 201 \ 200 \ 010 \quad 10 \ 210 \ 200 \ 010 \ 001 \quad 10 \ 300 \ 011 \\
 & S S - 26 a S S S S - 22 a S S S^2 - 44 a S S \\
 & \quad 100 \ 010 \quad 10 \ 300 \ 020 \ 100 \ 001 \quad 10 \ 300 \ 110 \ 010 \quad 10 \ 300 \ 110 \\
 & S S - 144 a S^2 S S - 72 a S S S S - 144 a S S \\
 & \quad 010 \ 001 \quad 11 \ 110 \ 101 \ 100 \quad 11 \ 200 \ 110 \ 020 \ 100 \quad 11 \ 200 \ 110 \\
 & S S - 144 a S S S S - 72 a S S^2 S - 36 a S^2 S \\
 & \quad 011 \ 100 \quad 11 \ 200 \ 110 \ 101 \ 010 \quad 11 \ 200 \ 110 \ 001 \quad 11 \ 200 \ 011 \ 010 \\
 & - 18 a S^2 S S + 24 a S S^4 + 192 a S S^3 S \\
 & \quad 11 \ 200 \ 020 \ 001 \quad 12 \ 021 \ 100 \quad 12 \ 111 \ 100 \ 010
 \end{aligned}$$

$$\begin{aligned}
 & + 96 \frac{a}{12} \frac{s}{120} \frac{s^3}{100} \frac{s}{001} + 144 \frac{a}{12} \frac{s}{201} \frac{s^2}{100} \frac{s^2}{010} + 288 \frac{a}{12} \frac{s}{210} \frac{s^2}{100} \frac{s}{010} \frac{s}{001} \\
 & + 96 \frac{a}{12} \frac{s}{300} \frac{s}{100} \frac{s^2}{010} \frac{s}{001} + 96 \frac{a}{13} \frac{s}{101} \frac{s}{020} \frac{s^3}{100} + 192 \frac{a}{13} \frac{s}{110} \frac{s}{011} \frac{s^3}{100} \\
 & + 576 \frac{a}{13} \frac{s}{110} \frac{s}{101} \frac{s^2}{100} \frac{s}{010} + 288 \frac{a}{13} \frac{s^2}{110} \frac{s^2}{100} \frac{s^2}{001} + 288 \frac{a}{13} \frac{s}{200} \frac{s}{011} \frac{s^2}{100} \\
 & \frac{s}{010} + 144 \frac{a}{13} \frac{s}{200} \frac{s}{020} \frac{s^2}{100} \frac{s}{001} + 288 \frac{a}{13} \frac{s}{200} \frac{s}{101} \frac{s}{100} \frac{s^2}{010} + 576 \frac{a}{13} \frac{s}{200} \\
 & \frac{s}{110} \frac{s}{100} \frac{s}{010} \frac{s}{001} + 72 \frac{a}{13} \frac{s^2}{200} \frac{s^2}{010} \frac{s}{001} - 240 \frac{a}{14} \frac{s}{011} \frac{s^4}{100} \frac{s}{010} - 120 \frac{a}{14} \\
 & \frac{s}{020} \frac{s^4}{100} \frac{s}{001} - 480 \frac{a}{14} \frac{s}{101} \frac{s^3}{100} \frac{s^2}{010} - 960 \frac{a}{14} \frac{s}{110} \frac{s}{100} \frac{s}{010} \frac{s}{001} \\
 & - 720 \frac{a}{14} \frac{s}{200} \frac{s^2}{100} \frac{s^2}{010} \frac{s}{001} + 720 \frac{s^4}{100} \frac{s^2}{010} \frac{s}{001} \\
 \\
 K_{331} = & a \frac{s}{1} \frac{s}{331} - 3 \frac{a}{2} \frac{s}{321} \frac{s}{010} - a \frac{s}{2} \frac{s}{330} \frac{s}{001} - 3 \frac{a}{231} \frac{s}{100} - 3 \frac{a}{3} \frac{s}{311} \frac{s}{020} - \\
 & 3 \frac{a}{3} \frac{s}{320} \frac{s}{011} - 9 \frac{a}{3} \frac{s}{221} \frac{s}{110} - 3 \frac{a}{3} \frac{s}{230} \frac{s}{101} - 3 \frac{a}{3} \frac{s}{131} \frac{s}{200} - a \frac{s}{4} \frac{s}{301} \frac{s}{030} - \\
 & 3 \frac{a}{4} \frac{s}{310} \frac{s}{021} - 9 \frac{a}{4} \frac{s}{211} \frac{s}{120} - 9 \frac{a}{4} \frac{s}{220} \frac{s}{111} - 9 \frac{a}{4} \frac{s}{121} \frac{s}{210} - 3 \frac{a}{4} \frac{s}{130} \frac{s}{201} \\
 & - a \frac{s}{4} \frac{s}{031} \frac{s}{300} + 6 \frac{a}{5} \frac{s}{311} \frac{s^2}{010} + 6 \frac{a}{5} \frac{s}{320} \frac{s}{010} \frac{s}{001} + 18 \frac{a}{5} \frac{s}{221} \frac{s}{010} \frac{s}{100} \\
 & + 6 \frac{a}{5} \frac{s}{230} \frac{s}{001} \frac{s}{100} + 6 \frac{a}{5} \frac{s}{131} \frac{s^2}{100} + 6 \frac{a}{6} \frac{s}{301} \frac{s}{020} \frac{s}{010} + 12 \frac{a}{6} \frac{s}{310} \frac{s}{011} \frac{s}{010} \\
 & + 6 \frac{a}{6} \frac{s}{310} \frac{s}{020} \frac{s}{001} + 36 \frac{a}{6} \frac{s}{211} \frac{s}{110} \frac{s}{010} + 18 \frac{a}{6} \frac{s}{220} \frac{s}{101} \frac{s}{010} + 18 \frac{a}{6} \frac{s}{220} \frac{s}{110} \frac{s}{001} \\
 & + 18 \frac{a}{6} \frac{s}{211} \frac{s}{020} \frac{s}{100} + 18 \frac{a}{6} \frac{s}{220} \frac{s}{011} \frac{s}{100} + 18 \frac{a}{6} \frac{s}{121} \frac{s}{200} \frac{s}{010} + 6 \frac{a}{6} \frac{s}{130} \frac{s}{200} \frac{s}{001} \\
 & + 36 \frac{a}{6} \frac{s}{121} \frac{s}{110} \frac{s}{100} + 12 \frac{a}{6} \frac{s}{130} \frac{s}{101} \frac{s}{100} + 6 \frac{a}{6} \frac{s}{031} \frac{s}{100} \frac{s}{200} + 36 \frac{a}{7} \frac{s}{111} \frac{s}{210} \frac{s}{010} \\
 & + 18 \frac{a}{7} \frac{s}{120} \frac{s}{201} \frac{s}{010} + 18 \frac{a}{7} \frac{s}{120} \frac{s}{210} \frac{s}{001} + 36 \frac{a}{7} \frac{s}{120} \frac{s}{111} \frac{s}{100} + 6 \frac{a}{7} \frac{s}{021} \frac{s}{300} \frac{s}{010} \\
 & + 2 \frac{a}{7} \frac{s}{030} \frac{s}{300} \frac{s}{001} + 18 \frac{a}{7} \frac{s}{021} \frac{s}{210} \frac{s}{100} + 6 \frac{a}{7} \frac{s}{030} \frac{s}{201} \frac{s}{100} + 6 \frac{a}{8} \frac{s}{300} \frac{s}{020} \frac{s}{011} \\
 & + 18 \frac{a}{8} \frac{s}{201} \frac{s}{020} \frac{s}{110} + 36 \frac{a}{8} \frac{s}{210} \frac{s}{011} \frac{s}{110} + 18 \frac{a}{8} \frac{s}{210} \frac{s}{020} \frac{s}{101} + 36 \frac{a}{8} \frac{s}{111} \frac{s^2}{110} \\
 & + 36 \frac{a}{8} \frac{s}{120} \frac{s}{110} \frac{s}{101} + 18 \frac{a}{8} \frac{s}{111} \frac{s}{020} \frac{s}{200} + 18 \frac{a}{8} \frac{s}{120} \frac{s}{011} \frac{s}{200} + 18 \frac{a}{8} \frac{s}{021} \frac{s}{110} \frac{s}{200}
 \end{aligned}$$

$$\begin{aligned}
 & + 6a \begin{matrix} S & S & S \\ 8 & 030 & 101 & 200 \end{matrix} - 18 \begin{matrix} a & S & S^3 \\ 9 & 301010 \end{matrix} - 18 \begin{matrix} a & S & S^3 & S^2 & S \\ 9 & 301 & 310 & 010 & 001 \end{matrix} \\
 & - 54a \begin{matrix} S & S^2 & S \\ 9 & 211 & 010 & 100 \end{matrix} - 54 \begin{matrix} a & S & S^3 \\ 9 & 220 & 010 & 100 \end{matrix} - 54 \begin{matrix} a & S & S & S^2 \\ 9 & 121 & 010 & 100 \end{matrix} - 18a \begin{matrix} S & S & S^2 \\ 9 & 130 & 001 & 100 \end{matrix} \\
 & - 6a \begin{matrix} S & S^3 \\ 9 & 031 & 100 \end{matrix} - 18 \begin{matrix} a & S & S & S^2 \\ 10 & 300 & 011 & 010 \end{matrix} - 18 \begin{matrix} a & S & S & S & S \\ 10 & 300 & 020 & 010 & 101 \end{matrix} - 54a \begin{matrix} S & S & S^2 \\ 10 & 201 & 110 & 010 \end{matrix} \\
 & - 54a \begin{matrix} S & S^2 & S^2 \\ 10 & 210 & 101 & 010 \end{matrix} - 108a \begin{matrix} S & S & S & S \\ 10 & 200 & 110 & 010 & 001 \end{matrix} - 54a \begin{matrix} S & S & S & S \\ 100201 & 020 & 010 & 100 \end{matrix} \\
 & - 108 \begin{matrix} a & S & S & S & S \\ 10 & 210 & 011 & 010 & 100 \end{matrix} - 54 \begin{matrix} a & S & S & S & S \\ 10 & 210 & 020 & 001 & 100 \end{matrix} - 54 \begin{matrix} a & S & S & S^2 \\ 10 & 111 & 200 & 010 \end{matrix} \\
 & - 54 \begin{matrix} a & S & S & S & S \\ 10 & 120 & 200 & 010 & 001 \end{matrix} - 216 \begin{matrix} a & S & S & S & S \\ 10 & 111 & 110 & 010 \end{matrix} - 108 \begin{matrix} a & S & S & S & S \\ 10 & 120 & 101 & 100 & 010 \end{matrix} \\
 & - 108 \begin{matrix} a & S & S & S & S \\ 10 & 120 & 110 & 100 & 001 \end{matrix} - 54 \begin{matrix} a & S & S & S^2 \\ 10 & 111 & 020 & 100 \end{matrix} - 54 \begin{matrix} a & S & S & S^2 \\ 10 & 120 & 011 & 100 \end{matrix} \\
 & - 54 \begin{matrix} a & S & S & S & S \\ 10 & 021 & 200 & 100 & 010 \end{matrix} \\
 & - 18 \begin{matrix} a & S & S & S & S \\ 10 & 030 & 200 & 100 & 001 \end{matrix} - 54 \begin{matrix} a & S & S & S^2 \\ 10 & 021 & 110 & 100 \end{matrix} - 18 \begin{matrix} a & S & S & S^2 \\ 10 & 030 & 101 & 100 \end{matrix} \\
 & - 108 \begin{matrix} a & S^2 & S & S \\ 11 & 110 & 101 & 010 \end{matrix} - 36 \begin{matrix} a & S^3 & S \\ 11 & 110 & 001 \end{matrix} - 54 \begin{matrix} a & S & S & S & S \\ 11 & 200 & 101 & 020 & 010 \end{matrix} \\
 & - 108 \begin{matrix} a & S & S & S & S \\ 11 & 200 & 110 & 011 & 010 \end{matrix} - 54 \begin{matrix} a & S & S & S & S \\ 11 & 200 & 110 & 020 & 001 \end{matrix} - 108 \begin{matrix} a & S \\ 11 & 011 \end{matrix} \\
 & S^4 \begin{matrix} S & S \\ 110 & 100 \end{matrix} - 108 \begin{matrix} a & S & S & S & S \\ 11 & 020 & 110 & 101 & 100 \end{matrix} - 54 \begin{matrix} a & S & S & S & S \\ 11 & 020 & 011 & 200 & 100 \end{matrix} \\
 & + 24 \begin{matrix} a & S & S^3 & S \\ 12 & 300 & 010 & 001 \end{matrix} + 72 \begin{matrix} a & S & S^3 & S \\ 12 & 201 & 010 & 100 \end{matrix} + 216 \begin{matrix} a & S & S^2 & S & S \\ 12 & 210 & 010 & 001 & 100 \end{matrix} \\
 & + 72 \begin{matrix} a & S & S & S^3 \\ 12 & 021 & 010 & 100 \end{matrix} + 24 \begin{matrix} a & S & S & S^3 \\ 12 & 030 & 001 & 100 \end{matrix} + 216 \begin{matrix} a & S & S^2 & S^2 \\ 12 & 111 & 100 & 010 \end{matrix} \\
 & + 216 \begin{matrix} a & S & S^2 & S & S \\ 12 & 120 & 100 & 010 & 001 \end{matrix} + 72 \begin{matrix} a & S & S & S^3 \\ 13 & 101 & 200 & 010 \end{matrix} + 216 \begin{matrix} a & S & S & S^2 \\ 13 & 110 & 200 & 010 \end{matrix} \\
 & S + 432 \begin{matrix} a & S & S & S^2 & S \\ 13 & 110 & 101 & 010 & 100 \end{matrix} + 432 \begin{matrix} a & S^2 & S & S & S \\ 13 & 110 & 010 & 001 & 100 \end{matrix} + 216 \begin{matrix} a & S \\ 13 & 200 \end{matrix} \\
 & S S^2 S + 216 \begin{matrix} a & S & S & S & S & S \\ 13 & 200 & 020 & 010 & 001 & 100 \end{matrix} + 432 \begin{matrix} a & S & S & S & S^2 \\ 13 & 011 & 110 & 010 & 100 \end{matrix} \\
 & + 216 \begin{matrix} a & S & S & S & S^2 \\ 13 & 020 & 101 & 010 & 100 \end{matrix} + 216 \begin{matrix} a & S & S & S & S^2 \\ 13 & 020 & 110 & 001 & 100 \end{matrix} + 72 \begin{matrix} a & S & S \\ 13 & 020 & 011 \end{matrix} \\
 & S^3 - 360 \begin{matrix} a & S & S^3 & S & S \\ 14 & 200 & 010 & 001 & 100 \end{matrix} - 360 \begin{matrix} a & S & S^3 & S^2 \\ 14 & 101 & 010 & 100 \end{matrix} - 1080a \begin{matrix} S & S^2 \\ 14 & 110 & 010 \end{matrix} \\
 & S S^2 - 360 \begin{matrix} a & S & S^2 & S^3 \\ 14 & 011 & 010 & 100 \end{matrix} - 360 \begin{matrix} a & S & S & S & S^3 \\ 14 & 020 & 010 & 001 & 100 \end{matrix} + 720 \begin{matrix} S^3 \\ 010 \end{matrix} \\
 & S S^3 \} \\
 & 001 \ 100 \}
 \end{aligned}$$

$$\begin{aligned}
 k_{322} = a_0 & \left\{ \begin{aligned} & a S - 2 a S S - 2 a S S - 3 a S S - a S S \\ & 1 \ 322 \quad 2 \ 312 \ 010 \quad 2 \ 321 \ 001 \quad 2 \ 222 \ 100 \quad 3 \ 302 \ 020 \\ & - 4 a S S - a S S - 6 a S S - 6 a S S - 3 a S S \\ & 3 \ 311 \ 011 \quad 3 \ 320 \ 002 \quad 3 \ 212 \ 110 \quad 3 \ 221 \ 101 \quad 3 \ 122 \ 200 \\ & - 2 a S S - 2 a S S - 3 a S S - 12 a S S - 3 a S S \\ & 4 \ 301 \ 021 \quad 4 \ 310 \ 012 \quad 4 \ 202 \ 120 \quad 4 \ 211 \ 111 \quad 4 \ 220 \ 102 \\ & - 6 a S S - 6 a S S - a S S + 2 a S S^2 + 8 a S S \\ & 4 \ 112 \ 210 \quad 4 \ 121 \ 201 \quad 4 \ 022 \ 300 \quad 5 \ 302 \ 010 \quad 5 \ 311 \ 010 \\ & S + 2 a S S^2 + 12 a S S S + 12 a S S S + 6 a S S^2 \\ & 001 \quad 5 \ 320 \ 001 \quad 5 \ 212 \ 010 \ 100 \quad 5 \ 221 \ 001 \ 100 \quad 5 \ 122 \ 100 \\ & + 8 a S S S + 4 a S S S + 4 a S S S + 8 a S S S \\ & 6 \ 301 \ 011 \ 010 \quad 6 \ 301 \ 020 \ 001 \quad 6 \ 310 \ 002 \ 010 \quad 6 \ 310 \ 011 \ 001 \\ & + 12 a S S S + 24 a S S S + 24 a S S S + 12 a S \\ & 6 \ 202 \ 110 \ 010 \quad 6 \ 211 \ 110 \ 001 \quad 6 \ 211 \ 101 \ 010 \quad 6 \ 220 \\ & S S + 6 a S S S + 24 a S S S + 6 a S S S \\ & 101 \ 001 \quad 6 \ 202 \ 020 \ 100 \quad 6 \ 211 \ 011 \ 100 \quad 6 \ 220 \ 002 \ 100 \\ & + 12 a S S S + 12 a S S S + 24 a S S S + 24 a S \\ & 6 \ 112 \ 200 \ 010 \quad 6 \ 121 \ 200 \ 001 \quad 6 \ 112 \ 110 \ 100 \quad 5 \ 121 \\ & S S + 6 a S S S + 12 a S S S + 24 a S S S \\ & 101 \ 100 \quad 6 \ 022 \ 100 \ 200 \quad 7 \ 102 \ 210 \ 010 \quad 7 \ 111 \ 201 \ 010 \\ & + 24 a S S S + 12 a S S S + 24 a S^2 S + 12 a S S \\ & 7 \ 111 \ 210 \ 001 \quad 7 \ 120 \ 201 \ 001 \quad 7 \ 111 \ 100 \quad 7 \ 120 \ 102 \\ & S + 4 a S S S + 4 a S S S + 12 a S S S + 12 a S \\ & 100 \quad 7 \ 012 \ 300 \ 010 \quad 7 \ 021 \ 300 \ 001 \quad 7 \ 012 \ 210 \ 100 \quad 7 \ 021 \\ & S S + 4 a S S^2 + 2 a S S S + 24 a S S S + 12 a S \\ & 201 \ 100 \quad 8 \ 300 \ 011 \quad 8 \ 300 \ 020 \ 002 \quad 8 \ 201 \ 011 \ 110 \quad 8 \ 201 \\ & S S + 12 a S S S + 24 a S S S + 12 a S S^2 + 48 a \\ & 020 \ 101 \quad 8 \ 210 \ 002 \ 110 \quad 8 \ 210 \ 011 \ 101 \quad 8 \ 102 \ 110 \quad 8 \\ & S S S + 12 a S S^2 + 6 a S S S + 24 a S S S \\ & 111 \ 110 \ 101 \quad 8 \ 120 \ 101 \quad 8 \ 102 \ 020 \ 200 \quad 8 \ 111 \ 011 \ 200 \\ & + 6 a S S S + 12 a S S S + 12 a S S S - 20 a S \\ & 8 \ 120 \ 002 \ 200 \quad 8 \ 012 \ 110 \ 200 \quad 8 \ 021 \ 101 \ 200 \quad 9 \ 301 \\ & S^2 S - 4 a S S S^2 - 18 a S S^2 S - 72 a S S S \\ & 010 \ 001 \quad 9 \ 310 \ 010 \ 001 \quad 9 \ 202 \ 010 \ 100 \quad 9 \ 211 \ 010 \ 001 \ 100 \\ & - 18 a S S^2 S - 36 a S S S - 36 a S S S^2 - 6 a S S^3 \\ & 9 \ 220 \ 001 \ 100 \quad 9 \ 112 \ 010 \ 100 \quad 9 \ 121 \ 001 \ 100 \quad 9 \ 022 \ 100 \\ & - 6 a S S S^2 - 24 a S S S S - 6 a S S S^2 \\ & 10 \ 300 \ 002 \ 010 \quad 10 \ 300 \ 011 \ 010 \ 001 \quad 10 \ 300 \ 020 \ 001 \\ & - 36 a S S S^2 - 72 a S S S S - 72 a S S S S \\ & 10 \ 201 \ 101 \ 010 \quad 10 \ 201 \ 110 \ 010 \ 001 \quad 10 \ 210 \ 101 \ 010 \ 001 \end{aligned} \right.
 \end{aligned}$$

$$\begin{aligned}
 & - 36 a s s s^2 - 72 a s s s s - 36 a s s s s - 36 a s s s s \\
 & \quad 10 210 110 001 \quad 10 201 011 010 100 \quad 10 210 002 010 100 \\
 & - 72 a s s s s - 36 a s s s s - 18 a s s s^2 \\
 & \quad 10 210 011 001 100 \quad 10 201 020 001 100 \quad 10 102 200 010 \\
 & - 72 a s s s s - 18 a s s s^2 - 72 a s s s s \\
 & \quad 10 111 200 010 001 \quad 10 120 200 001 \quad 10 102 110 100 010 \\
 & - 144 a s s s s - 144 a s s s s - 72 a s s s s \\
 & \quad 10 111 101 100 010 \quad 10 111 110 100 001 \quad 10 120 101 100 \\
 & s - 18 a s s s^2 - 72 a s s s^2 - 18 a s s s^2 \\
 & \quad 001 \quad 10 102 020 100 \quad 10 111 011 100 \quad 10 120 002 100 \\
 & - 36 a s s s s - 36 a s s s s - 36 a s s s^2 \\
 & \quad 10 012 200 100 010 \quad 10 021 200 100 001 \quad 10 012 110 100 \\
 & - 36 a s s s^2 - 72 a s s^2 s - 72 a s^2 s s \\
 & \quad 10 021 101 100 \quad 11 110 101 010 \quad 11 110 101 001 \\
 & - 72 a s s s s - 36 a s s s s - 36 a s s s s \\
 & \quad 11 200 101 011 010 \quad 11 200 110 002 010 \quad 11 200 101 020 \\
 & s - 72 a s s s s - 36 a s s^2 s - 144 a s s s s \\
 & \quad 001 \quad 11 200 110 011 001 \quad 11 002 110 100 \quad 11 011 110 101 \\
 & s - 36 a s s^2 s - 36 a s^2 s s - 18 a s s s s \\
 & \quad 100 \quad 11 020 101 100 \quad 11 011 200 100 \quad 11 020 002 200 100 \\
 & + 24 a s s^2 s^2 + 144 a s s^2 s s + 144 a s s s^2 s \\
 & \quad 12 300 010 001 \quad 12 201 010 001 100 \quad 12 210 010 001 100 \\
 & + 48 a s s s^3 + 48 a s s s^3 + 72 a s s^2 s^2 \\
 & \quad 12 012 010 100 \quad 12 021 001 100 \quad 12 102 100 010 \\
 & + 288 a s s^2 s s + 72 a s s^2 s^2 + 144 a s s s^2 s \\
 & \quad 12 111 100 010 001 \quad 12 120 100 001 \quad 13 101 200 010 001 \\
 & + 144 a s s s s^2 + 144 a s^2 s^2 s + 476 a s s s s \\
 & \quad 13 110 200 010 001 \quad 13 101 010 100 \quad 13 110 101 010 \\
 & s s + 144 a s^2 s s s + 72 a s s s^2 s + 288 a \\
 & \quad 001 100 \quad 13 110 010 001 100 \quad 13 200 002 010 100 \quad 13 \\
 & s s s s s + 72 a s s s^2 s + 144 a s s s s^2 \\
 & \quad 200 011 010 100 001 \quad 13 200 020 001 100 \quad 13 002 110 010 100 \\
 & + 288 a s s s s^2 + 288 a s s s s^2 + 144 a s s s \\
 & \quad 13 011 110 001 100 \quad 13 011 101 010 100 \quad 13 020 101 \\
 & s s^2 + 48 a s^2 s^3 + 24 a s s s^3 - 360 a s s^2 s^2 s \\
 & \quad 001 100 \quad 13 011 100 \quad 13 020 002 100 \quad 14 200 010 001 100 \\
 & - 720 a s s^2 s s^2 - 720 a s s s^2 s^2 - 120 a s s^2 \\
 & \quad 14 101 010 001 100 \quad 14 110 010 001 100 \quad 14 002 010 \\
 & s^3 - 480 a s s s s^3 - 120 a s s^2 s^3 + 720 s^2 s^2 s^3 \} \\
 & \quad 100 \quad 14 011 010 001 100 \quad 14 020 001 100 \quad 010 001 100
 \end{aligned}$$

TABLE 1

Give the Number of tripartitions of the tripartite number
(a b c = w = 7) into v parts

abc v	700	610	520	511	430	421	331	322
1	1	1	1	1	1	1	1	1
2	3	6	8	11	9	14	15	17
3	4	9	15	21	18	33	36	46
4	3	7	12	17	16	29	34	44
5	2	4	7	9	9	15	17	22
6	1	2	3	4	3	5	5	6
7	1	1	1	1	1	1	1	1

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