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Locally A-optimal Design for Poisson Regression Model with Two Parameters

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Keywords

Poisson regression model; Local optimal design; Fisher information matrix A-optimal design; Equivalence theorem

Abstract

An algebraic method for constructing A-optimal design for two parameters Poisson regression model including intercept parameter is presented. As we know, the Fisher information matrix depends on the unknown parameters of the model. In such a case, an experimenter must take the strategy of discovering local optimal designs, that is, first predict the best value for the parameters and then compute the optimal designs. In this article, we obtained locally A-optimal designs based on two- and three- point settings. Support points and weights are calculated numerically through *Mathematica* software. Also, the necessary and sufficient conditions of this optimality criterion are confirmed through the equivalence theorem.

Mathematical Subject Classification: 62J12,62B15

1. Introduction

Nelder and Wedderburn (1972) developed the generalized linear model (GLM) which is regarded as an extension of standard linear regression in that it allows continuous or discrete data from one-parameter exponential family distributions to be paired with explanatory variables using appropriate link functions. Generalized linear models include several types such as Poisson, Gamma, Logistic models among others. Generalized linear models are generally utilized in studies when the responses are categorical type, however in the case of count data, the investigator proceeds to the Poisson model. These models offer a fascinating and satisfying modeling experience for experimenters. The analysis and inference methods for these models are well-established (McCullough and Nelder, 1989).

The goal for developing an optimal design is to get statistical conclusions about the quantities of interest by appropriately selecting the control variable. The control variable values are selected to minimize the variability of the estimators of the unknown parameters included in the regression model. The foundation work on optimal design was laid by Kiefer (1959) and Kiefer and Wolfowitz (1959). There is substantial research on the topic of optimal design for classical linear models with normal errors. The problem is quite simple since the information matrix is independent of the unknown parameters but in case of generalized linear models, it depends on unknown parameters. The difficulty with designing experiments for such a model is that one is seeking for the optimal design with the goal of estimating the unknown parameters, but one must

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first know the parameters. Therefore, the optimal design cannot be found without a prior knowledge of the parameters. One method is the so-called local optimality, which was introduced by Chernoff (1953). This method seeks to generate an optimal design at a given parameter value (best initial guess). The experimenter can choose the "best initial guess" value by the help of expert opinion or historical data from the review of literature.

An algebraic approach for constructing A-optimal designs for two parameters generalized linear models is presented by Min Yang (2008). For the main effects only model, the D-optimality of their design was proven analytically by Russell et al. (2009) even for larger dimensions. Ying Schmidt and Schwabe (2017) extended Russell et al.'s conclusion for higher dimension regression to a considerably larger class of additive models. Freise et al. (2021) characterized D-optimal designs in two dimensional Poisson regression model with interaction. Idais (2021) obtained D-, A-, and Kiefer's Φ_k -criteria optimality for vertex-type designs for both Gamma & Poisson models. Niaparast et al. (2023) developed a new version of the equivalence theorem for the E criteria in the Poisson regression model with random effects. Recently, R-optimal design for Poisson regression model for two parameters and linear regression model with two variables discussed by Biswal (2024). Also necessary and sufficient conditions are verified through the general equivalence theorem. In this context, the present article aims to construct locally A-optimal designs for the Poisson regression model with two parameters including the intercept parameter is supported by the design points. The necessary and sufficient condition is confirmed by using the corresponding equivalence theorem.

The rest of the article is organized as follows. Section 2 includes the preliminary information. Section 3 presents a brief discussion on A-optimal designs & general equivalence theorem. In Section 4, locally A-optimal designs for the Poisson regression model with two parameters are discussed. A numerical investigation for optimality verification is added in section5. Finally, the article is concluded with some discussions and conclusions in Section 6.

2. Preliminaries

Let us consider an experiment where the i^{th} observation on a response variable, y_i, has a Poisson distribution, with a rate λ_i dependent on q-independent covariates $\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_q$ through the log-linear model i.e.

$$\ln(\lambda_i) = \eta_i = \mathbf{g}(\mathbf{z}_i)^T \boldsymbol{\beta}, \; i = 1,2,.., n \tag{1}$$

where $\mathbf{z}_i = (z_1, z_2, ..., z_q)$, $g(\mathbf{z}_i) = (1, z_i^T)^T$, $\boldsymbol{\beta} = \beta_1, \beta_2, ..., \beta_q$ are q-dimensional vectors of the unknown parameters. For the model (1) and log link, the Fisher information matrix is $q \times q$ dimension at $\mathbf{z}$ and $\boldsymbol{\beta}$ can be defined as

$$\mathbf{M}(\mathbf{z}, \boldsymbol{\beta}) = \kappa\, \mathbf{g}(\mathbf{z})\mathbf{g}^T(\mathbf{z}) \tag{2}$$

where $\kappa = e^{\eta_i}$ is the log-link function or intensity function (see Freise et. al, 2021).

To obtain the A-optimal design for the model (1), consider the approximate design $\xi \in \Delta$ (Δ the set of all approximate designs) of the form

$$\xi = \begin{Bmatrix} \mathbf{z}_1 & \mathbf{z}_2 & ... & \mathbf{z}_q \\ \omega_1 & \omega_2 & ... & \omega_q \end{Bmatrix}, \text{ and} \sum_{i=1}^{q} \omega_i = 1 \tag{3}$$

where $\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_q \in \Omega$ ($\Omega \subset R^q$) are the 'q' distinct points and ω_i is the weight associated with the point $\mathbf{z}_i$ for $i=1, 2, ..., q$. For the model (1), the Fisher information matrix of a design ξ at parameter $\boldsymbol{\beta}$ is defined as

$$\mathbf{M}(\xi, \boldsymbol{\beta}) = \sum_{i=1}^{q} \omega_i \mathbf{M}(\mathbf{z}_i, \boldsymbol{\beta}) \tag{4}$$

For further information, one can refer to pg152, (see Russel, 2018).

3. A-optimal design & General Equivalence theorem

Definition 1: A design $\xi^* \in \Delta$ with an information matrix $\mathbf{M}(\xi)$ for model (1) is called A-optimal design if it minimizes Trace $(\mathbf{M}^{-1}(\xi))$ over $\boldsymbol{\Omega}$.

Definition 2: A minimum support design for any regression model having q parameters is supported on exactly q distinct support points (see Goos and Vandebroek, 2001).

The following equivalence theorem established by Fedorov (1971) provides the necessary and sufficient conditions for the determination of A-optimal design over the simplex region Ω.

Theorem 1: A design $\xi^* \in \Delta$ is A-optimal for model (1) if and only if

$$\underset{\mathbf{z} \in \Omega}{\text{Max}} \ \zeta(\mathbf{z}, \xi^*) = \text{Trace}(\mathbf{M}^{-1}(\xi^*)) \tag{5}$$

where $\zeta(\mathbf{z}, \xi) = \kappa \mathbf{g}(\mathbf{z}) \mathbf{M}^{-2}(\xi) \mathbf{g}^T(\mathbf{z})$. Moreover, the supremum exists at the support point of ξ^*.

4. A-optimal designs for two parameters

In this section, locally A-optimal designs for the model (1) i.e. $\mathbf{g}^T(\mathbf{z}) = \beta_0 + \beta_1 z > 0$ for all $z \in R$. Here, we restrict our search to two-, and three-support points design by considering discrete values of β_0, β_1 in the randomly chosen intervals [1, 15] & [1, 10].

4.1. Designs based on two support points

Let us consider a 2-point design ξ of the form

$$\xi = \begin{Bmatrix} x_1 & x_2 \\ \omega & 1-\omega \end{Bmatrix} \text{where } 0 < \omega < 1. \tag{6}$$

Theorem 4.1.1 The design ξ^* that assigns a weight ω^* to the point x_1^* and $1-\omega^*$ to the point x_2^* in Ω is an A-optimal design where x_1^*, x_2^*, and ω^* are given in Table 2 (Appendix).

Proof. The information matrix for the model (6) at the two-point design ξ defined in (4) is given by

$$\mathbf{M}(\xi)=\begin{bmatrix} e^{\beta_0+x_2\beta_1}(1-\omega)+e^{\beta_0+x_1\beta_1}\omega & x_2e^{\beta_0+x_2\beta_1}(1-\omega)+x_1e^{\beta_0+x_1\beta_1}\omega \\ x_2e^{\beta_0+x_2\beta_1}(1-\omega)+x_1e^{\beta_0+x_1\beta_1}\omega & x_2^2e^{\beta_0+x_2\beta_1}(1-\omega)+x_1^2e^{\beta_0+x_1\beta_1}\omega \end{bmatrix}. \quad (7)$$

The inverse of the above Fisher information matrix is given by

$$\mathbf{M}^{-1}(\xi)=\begin{bmatrix} \frac{e^{-\beta_0-(x_1+x_2)\beta_1}(x_2^2e^{x_2\beta_1}(-1+\omega)-x_1^2e^{x_1\beta_1}\omega)}{(x_1-x_2)^2(-1+\omega)\omega} & \frac{e^{-\beta_0-(x_1+x_2)\beta_1}(-x_2e^{x_2\beta_1}(-1+\omega)+x_1e^{x_1\beta_1}\omega)}{(x_1-x_2)^2(-1+\omega)\omega} \\ \frac{e^{-\beta_0-(x_1+x_2)\beta_1}(-x_2e^{x_2\beta_1}(-1+\omega)+x_1e^{x_1\beta_1}\omega)}{(x_1-x_2)^2(-1+\omega)\omega} & \frac{e^{-\beta_0-(x_1+x_2)\beta_1}(e^{x_2\beta_1}(-1+\omega)-e^{x_1\beta_1}\omega)}{(x_1-x_2)^2(-1+\omega)\omega} \end{bmatrix}. \quad (8)$$

Using (8), we obtain the trace function $\Phi(\xi)$ i.e.

$$\Phi(\xi)=\frac{e^{-\beta_0-(x_1+x_2)\beta_1}((1+x_2^2)e^{x_2\beta_1}(-1+\omega)-(1+x_1^2)e^{x_1\beta_1}\omega)}{(x_1-x_2)^2(-1+\omega)\omega}\ . \quad (9)$$

Now, the problem is to minimize the function $\Phi(\xi)$ with respect to x_1, x_2, and ω for given values of β_0 and β_1. This is done using the "*Nminimize*" function of *Wolfram Mathematica7.0* software and getting the optimal values x_1^*, x_2^*, and ω^*. The numerical values of x_1^*, x_2^*, and ω^* are given in Table 2 (Appendix).

The necessary and sufficient condition of the locally A-optimal design i.e. $\kappa\mathbf{g}(\mathbf{z})\mathbf{M}^{-2}(\xi)\mathbf{g}^T(\mathbf{z})\le \text{Trace}(\mathbf{M}^{-1}(\xi))$ is verified by using the equivalence theorem which is as follows:

$$\zeta(\mathbf{z},\xi^*)=\frac{d1-d2+d3+d4}{(x_1-x_2)^4(-1+\omega)^2\omega^2} \quad (10)$$

where $d1=e^{-\beta_0+\beta_1}(-2(x_1+x_2)+z)(x_2^4e^{2x_2\beta_1}(-1+\omega)^2+x_1^4e^{2x_1\beta_1}\omega^2-2x_2^3e^{2x_2\beta_1}(-1+\omega)^2z$

$d2=2x_1^3e^{2x_1\beta_1}\omega^2z+2x_1\omega(e^{(x_1+x_2)\beta_1}(-1+\omega)-e^{2x_1\beta_1}\omega)z+(e^{x_2\beta_1}(-1+\omega)z-e^{x_1\beta_1}\omega z)^2$

$d3=x_1^2e^{2x_1\beta_1}\omega^2(1+z^2)+2x_2e^{x_2\beta_1}(-1+\omega)(-e^{x_2\beta_1}(-1+\omega)z+e^{x_1\beta_1}\omega(x_1-z)(-1+x_1z))$

$d4=x_2^2e^{x_2\beta_1}(-1+\omega)(-2x_1e^{x_1\beta_1}\omega(x_1-z)+e^{x_2\beta_1}(-1+\omega)(1+z^2)))$.

Replacing the numerical values of x_1^*, x_2^*, and ω^* in (10) and we find $\underset{\mathbf{z}\in\Omega}{\text{Max}}\ \zeta(\mathbf{z},\xi^*)=$ Trace $(\mathbf{M}^{-1}(\xi^*))$. Thus the necessary and sufficient condition of the equivalence theorem is established. This proves Theorem 4.1.1. □

4.2. Designs based on three support points

Let us consider a 3-point design ξ of the form

$$\xi=\begin{Bmatrix} x_1 & x_2 & x_3 \\ \omega & (1-2\omega) & \omega \end{Bmatrix} \text{ where } 0<\omega<1. \quad (11)$$

Theorem 4.2.1 The design ξ^* that assigns a weight ω^* to the point x_1^*, $(1-2\omega^*)$ to the point x_2^*, and ω^* to the point x_3^* in Ω is an A-optimal design are given in Table 3 (Appendix).

Proof. Using (11), the information matrix for the model (1) at the three-point design ξ will be

$$\mathbf{M}(\xi)=\begin{bmatrix} e^{\beta_0+x_2\beta_1}(1-2\omega)+e^{\beta_0+x_1\beta_1}\omega+e^{\beta_0+x_3\beta_1}\omega & x_2e^{\beta_0+x_2\beta_1}(1-2\omega)+x_1e^{\beta_0+x_1\beta_1}\omega+x_1e^{\beta_0+x_3\beta_1}\omega) \\ x_2e^{\beta_0+x_2\beta_1}(1-2\omega)+x_1e^{\beta_0+x_1\beta_1}\omega+x_1e^{\beta_0+x_3\beta_1}\omega) & x_2^2e^{\beta_0+x_2\beta_1}(1-2\omega)+x_1^2e^{\beta_0+x_1\beta_1}\omega+x_1^2e^{\beta_0+x_3\beta_1}\omega) \end{bmatrix}.$$

The inverse of the above information matrix is given by

$$\mathbf{M}^{-1}(\xi)=\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{12}$$

with

$$M_{11}=\frac{e^{-\beta_0}(x_2^2e^{x_2\beta_1}(1-2\omega)+x_1^2e^{x_1\beta_1}\omega+x_3^2e^{x_3\beta_1}\omega)}{-(x_2e^{x_2\beta_1}(1-2\omega)+x_1e^{x_1\beta_1}\omega+x_3e^{x_3\beta_1}\omega)^2+(e^{x_2\beta_1}(1-2\omega)+e^{x_1\beta_1}\omega+e^{x_3\beta_1}\omega)(x_2^2e^{x_2\beta_1}(1-2\omega)+x_1^2e^{x_1\beta_1}\omega+x_3^2e^{x_3\beta_1}\omega)}$$

$$M_{12}=M_{21}=\frac{e^{-\beta_0}(-(x_1e^{x_1\beta_1}+x_3e^{x_3\beta_1}\omega)+x_2e^{x_2\beta_1}(-1+2\omega))}{-(x_2e^{x_2\beta_1}(1-2\omega)+x_1e^{x_1\beta_1}\omega+x_3e^{x_3\beta_1}\omega)^2+(e^{x_2\beta_1}(1-2\omega)+e^{x_1\beta_1}\omega+e^{x_3\beta_1}\omega)(x_2^2e^{x_2\beta_1}(1-2\omega)+x_1^2e^{x_1\beta_1}\omega+x_3^2e^{x_3\beta_1}\omega)}$$

$$M_{22}=\frac{e^{-\beta_0}(e^{x_2\beta_1}(1-2\omega)+e^{x_1\beta_1}\omega+e^{x_3\beta_1}\omega)}{-(x_2e^{x_2\beta_1}(1-2\omega)+x_1e^{x_1\beta_1}\omega+x_3e^{x_3\beta_1}\omega)^2+(e^{x_2\beta_1}(1-2\omega)+e^{x_1\beta_1}\omega+e^{x_3\beta_1}\omega)(x_2^2e^{x_2\beta_1}(1-2\omega)+x_1^2e^{x_1\beta_1}\omega+x_3^2e^{x_3\beta_1}\omega)}.$$

Using (12), we obtain the trace function $\Phi(\xi)$ i.e.

$$\Phi(\xi)=\frac{(e^{-\beta_0}(-(1+x_1^2)e^{x_1\beta_1}\omega-(1+x_3^2)2e^{x_3\beta_1}\omega+(1+x_2^2)e^{x_2\beta_1}(-1+2\omega))}{\omega(2x_1x_3e^{(x_1+x_3)\beta_1}\omega-x_3^2e^{(x_1+x_3)\beta_1}\omega-2x_1x_2e^{(x_1+x_2)\beta_1}(-1+2\omega)+e^{x_2\beta_1}(x_2^2e^{x_1\beta_1}+(x_2-x_3)^2e^{x_3\beta_1})(-1+2\omega)+x_1^2e^{x_1\beta_1}(-e^{x_3\beta_1}\omega+e^{x_2\beta_1}(-1+2\omega))}. \tag{13}$$

Next, the problem is need to minimize the function $\Phi(\xi)$ with respect to x_1, x_2, x_3 and ω for given values of β_0 and β_1. This is achieved by using the "*Nminimize*" function of *Wolfram Mathematica7.0* software and getting the optimal values x_1^*, x_2^*, x_3^* and ω^*. The numerical values of x_1^*, x_2^*, x_3^* and ω^* are given in Table 3 (Appendix).

The necessary and sufficient condition of the locally A-optimal design i.e. $\kappa\mathbf{g}(\mathbf{z})\mathbf{M}^{-2}(\xi)\mathbf{g}^T(\mathbf{z})\le \mathrm{Trace}(\mathbf{M}^{-1}(\xi))$ is verified by using the equivalence theorem which is as follows:

$$\zeta(\mathbf{z},\xi^*)=\frac{h1-h2+h3+h4+h5+h6-h7+h8}{h9+h10} \tag{14}$$

where

$$h1=(e^{-\beta_0+\beta_1 z}(x_2^4e^{2x_2\beta_1}(-1+2\omega)^2+x_1^4e^{2x_1\beta_1}\omega^2+x_3^2e^{2x_3\beta_1}\omega^2+x_3^4e^{2x_3\beta_1}\omega^2$$

$$h2=-2x_2^3e^{2x_2\beta_1}(-1+2\omega)^2z-2x_1^3e^{2x_1\beta_1}\omega^2z-2x_3e^{2x_3\beta_1}\omega^2z-2x_3^3e^{2x_3\beta_1}\omega^2z-2x_3e^{(x_1+x_3)\beta_1}\omega^2z$$

$$h3=+2e^{(x_2+x_3)\beta_1}\omega(-1+2\omega)(x_3-z)z+e^{2x_2\beta_1}z^2-4e^{2x_2\beta_1}\omega z^2+e^{2x_1\beta_1}\omega^2z^2+4e^{2x_2\beta_1}\omega^2z^2$$

$$h4 = +e^{2x_3\beta_1}\omega^2 z^2 + x_3^2 e^{2x_3\beta_1}\omega^2 z^2 + 2e^{(x_1+x_3)\beta_1}\omega^2 z^2 - 2e^{(x_1+x_2)\beta_1}\omega(-1+2\omega)z^2$$
$$h5 = +2x_2 e^{x_2\beta_1}(-1+2\omega)(-x_1 e^{x_1\beta_1}\omega + e^{x_2\beta_1}z + e^{x_1\beta_1}\omega z + x_1^2 e^{x_1\beta_1}\omega z - 2e^{x_2\beta_1}\omega z - x_1 e^{x_1\beta_1}\omega z^2$$
$$h6 = +e^{x_3\beta_1}\omega(x_3 - z)(-1 + x_3 z)) + x_1^2 e^{x_1\beta_1}\omega^2(2x_3 e^{x_3\beta_1}(x_3 - z) + e^{x_1\beta_1}(1+z^2))$$
$$h7 = -2x_1 e^{x_1\beta_1}\omega(x_3^2 e^{x_3\beta_1}\omega z + (e^{x_2\beta_1}(-1+2\omega) + e^{x_1\beta_1}\omega + e^{x_3\beta_1}\omega)z - x_3 e^{x_3\beta_1}\omega(1+z^2))$$
$$h8 = +x_2^2 e^{x_2\beta_1}(-1+2\omega)(-2x_1 e^{x_1\beta_1}\omega(x_1 - z) - 2x_3 e^{x_3\beta_1}\omega(x_3 - z) + e^{x_2\beta_1}(-1+2\omega)(1+z^2))))$$
$$h9 = (\omega^2(2x_1 x_3 e^{(x_1+x_3)\beta_1}\omega - x_3^2 e^{(x_1+x_3)\beta_1}\omega - 2x_1 x_2 e^{(x_1+x_2)\beta_1}(-1+2\omega)$$
$$h10 = + e^{x_2\beta_1}(x_2^2 e^{x_1\beta_1} + (x_2 - x_3)^2 e^{x_3\beta_1})(-1+2\omega) + x_1^2 e^{x_1\beta_1}(-e^{x_3\beta_1}\omega + e^{x_2\beta_1}(-1+2\omega)))^2).$$

Replacing the numerical values of x_1^*, x_2^*, x_3^* and ω^* in (14) and we find $\underset{\mathbf{z}\in\Omega}{\text{Max}}\ \zeta(\mathbf{z},\xi^*) =$ Trace $(\mathbf{M}^{-1}(\xi^*))$. Thus the necessary and sufficient condition of the equivalence theorem is established. This proves Theorem4.2.1. □

5. Numerical Investigation for Optimality verification

We used an example to verify the A-optimality of the above-mentioned design. Let us examine a design with two support points. Choose any one design from Table-2, say $\xi = \begin{Bmatrix} 0.4932 & 1 \\ 0.8182 & 0.1818 \end{Bmatrix}$ with $\beta_0 = 1, \beta_1 = 5$. Here, we find the Trace of $(\mathbf{M}^{-1}(\xi))$ is **0.3633**. As mentioned in Theorem-1 i.e. supremum or equality holds at the support points which are verified in the Table 1. Similar way, one can examine the A-optimality for three-support points also.

Table 1. Trace value of different support points

Support points	Trace of $(\mathbf{M}^{-1}(\xi))$
1	**0.3630**
0.4932	**0.3633**
0.234	0.2701
0.15	0.2257
0.1035	0.2020
0.05	0.1762
0.0025	0.1551

6. Conclusion

This article uses two- and three-support point designs to find locally A-optimal designs for a two-parameter Poisson regression model with a log link function. The support points of the optimal designs, as well as the weights assigned to these points, are numerically calculated using Wolfram Mathematica7.0's "*Nminimize*" function, while the necessary and sufficient condition of A-optimality, i.e. the equivalence theorem, is also established at the support point of the A-optimal design using Wolfram Mathematica7.0 software. Tables 2 and 3 (Appendix) includes a catalog of support points as well as the weight allocated to each support point corresponding to A-optimal designs. From Table 2 and Table 3, we find that the support points lie in the first quadrant of the two-dimensional space. Further, we observe that one of the support points always meet the boundary region associated with less weight as compared to other. Limitation of local optimality refers to a solution that is ideal with in small localized region of the solution space but may not be best overall (i.e. globally optimal). Russell et al. (2009) have obtained globally D-optimal designs for the Poisson regression model. In this article, it is assumed that equal weights are assigned to each of the design points.

An interesting research problem is to investigate globally A-optimal designs for the same model. This shall be an interesting and challenging research problem as the weights assigned to each design point may not be the same. To get global optimal design is always a challenging task; hence more study is needed in this area because the equivalence theorem is not valid for many discrete values of the unknown parameters. We look forward to exploring this open problem in future research. Bayesian optimization might be the future investigation of this subject. Furthermore, this concept may be extended to find various optimum designs for the Poisson regression model with more than two parameters for future Study.

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Appendix

Two support points design: Table-2 provides locally A-optimal designs is for $\boldsymbol{\beta} = (\beta_0, \beta_1)^T$ where $\beta_0, \beta_1 \in [1,15]$ with region space [0,1].

Table-2

$\boldsymbol{\beta}$	$\beta_0 = 1, \beta_1 = 1$	$\beta_0 = 1, \beta_1 = 2$	$\beta_0 = 1, \beta_1 = 3$	$\beta_0 = 1, \beta_1 = 4$
$\mathbf{z}$ $\boldsymbol{\omega}$	$\begin{Bmatrix} 1 & 0 \\ 0.3001 & 0.6999 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.2064 & 0.7936 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1750 \\ 0.1723 & 0.8277 \end{Bmatrix}$	$\begin{Bmatrix} 0.3708 & 1 \\ 0.8235 & 0.1765 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 1, \beta_1 = 5$	$\beta_0 = 1, \beta_1 = 6$	$\beta_0 = 1, \beta_1 = 7$	$\beta_0 = 1, \beta_1 = 8$
$\mathbf{z}$ $\boldsymbol{\omega}$	$\begin{Bmatrix} 0.4932 & 1 \\ 0.8182 & 0.1818 \end{Bmatrix}$	$\begin{Bmatrix} 0.5763 & 1 \\ 0.8136 & 0.1864 \end{Bmatrix}$	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8099 & 0.1991 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.8069 & 0.1931 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 1, \beta_1 = 9$	$\beta_0 = 1, \beta_1 = 10$	$\beta_0 = 1, \beta_1 = 11$	$\beta_0 = 1, \beta_1 = 12$
$\mathbf{z}$ $\boldsymbol{\omega}$	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8045 & 0.1955 \end{Bmatrix}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8024 & 0.1976 \end{Bmatrix}$	$\begin{Bmatrix} 0.7679 & 1 \\ 0.8008 & 0.1992 \end{Bmatrix}$	$\begin{Bmatrix} 0.7871 & 1 \\ 0.7992 & 0.2008 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 1, \beta_1 = 13$	$\beta_0 = 1, \beta_1 = 14$	$\beta_0 = 1, \beta_1 = 15$	$\beta_0 = 2, \beta_1 = 1$
$\mathbf{z}$ $\boldsymbol{\omega}$	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$	$\begin{Bmatrix} 0.8175 & 1 \\ 0.7970 & 0.2030 \end{Bmatrix}$	$\begin{Bmatrix} 0.8296 & 1 \\ 0.7961 & 0.2039 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.3001 & 0.6999 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 2, \beta_1 = 2$	$\beta_0 = 2, \beta_1 = 3$	$\beta_0 = 2, \beta_1 = 4$	$\beta_0 = 2, \beta_1 = 5$
$\mathbf{z}$ $\boldsymbol{\omega}$	$\begin{Bmatrix} 1 & 0 \\ 0.2064 & 0.7936 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1750 \\ 0.1723 & 0.8277 \end{Bmatrix}$	$\begin{Bmatrix} 0.3708 & 1 \\ 0.8235 & 0.1765 \end{Bmatrix}$	$\begin{Bmatrix} 0.4932 & 1 \\ 0.8182 & 0.1818 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 2, \beta_1 = 6$	$\beta_0 = 2, \beta_1 = 7$	$\beta_0 = 2, \beta_1 = 8$	$\beta_0 = 2, \beta_1 = 9$
$\mathbf{z}$ $\boldsymbol{\omega}$	$\begin{Bmatrix} 0.5763 & 1 \\ 0.8136 & 0.1864 \end{Bmatrix}$	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8099 & 0.1991 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.8069 & 0.1931 \end{Bmatrix}$	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8045 & 0.1955 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 2, \beta_1 = 10$	$\beta_0 = 2, \beta_1 = 11$	$\beta_0 = 2, \beta_1 = 12$	$\beta_0 = 2, \beta_1 = 13$
$\mathbf{z}$ $\boldsymbol{\omega}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8024 & 0.1976 \end{Bmatrix}$	$\begin{Bmatrix} 0.7679 & 1 \\ 0.8008 & 0.1992 \end{Bmatrix}$	$\begin{Bmatrix} 0.7871 & 1 \\ 0.7992 & 0.2008 \end{Bmatrix}$	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 2, \beta_1 = 14$	$\beta_0 = 2, \beta_1 = 15$	$\beta_0 = 3, \beta_1 = 1$	$\beta_0 = 3, \beta_1 = 2$
$\mathbf{z}$ $\boldsymbol{\omega}$	$\begin{Bmatrix} 0.8175 & 1 \\ 0.7970 & 0.2030 \end{Bmatrix}$	$\begin{Bmatrix} 0.8304 & 0.9999 \\ 0.7829 & 0.2121 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.3001 & 0.6999 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.2064 & 0.7936 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 3, \beta_1 = 3$	$\beta_0 = 3, \beta_1 = 4$	$\beta_0 = 3, \beta_1 = 5$	$\beta_0 = 3, \beta_1 = 6$
$\mathbf{z}$ $\boldsymbol{\omega}$	$\begin{Bmatrix} 1 & 0.1749 \\ 0.1723 & 0.8277 \end{Bmatrix}$	$\begin{Bmatrix} 0.3708 & 1 \\ 0.8235 & 0.1765 \end{Bmatrix}$	$\begin{Bmatrix} 0.4932 & 1 \\ 0.8182 & 0.1818 \end{Bmatrix}$	$\begin{Bmatrix} 0.5763 & 1 \\ 0.8136 & 0.1864 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 3, \beta_1 = 7$	$\beta_0 = 3, \beta_1 = 8$	$\beta_0 = 3, \beta_1 = 9$	$\beta_0 = 3, \beta_1 = 10$
$\mathbf{z}$ $\boldsymbol{\omega}$	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8099 & 0.1991 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.8069 & 0.1931 \end{Bmatrix}$	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8045 & 0.1955 \end{Bmatrix}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8024 & 0.1976 \end{Bmatrix}$

β	$\beta_0=3,\beta_1=11$	$\beta_0=3,\beta_1=12$	$\beta_0=3,\beta_1=13$	$\beta_0=3,\beta_1=14$
z **ω**	$\begin{Bmatrix}0.7679 & 1\\ 0.8008 & 0.1992\end{Bmatrix}$	$\begin{Bmatrix}0.7871 & 1\\ 0.7992 & 0.2008\end{Bmatrix}$	$\begin{Bmatrix}0.8035 & 0.9999\\ 0.7981 & 0.2019\end{Bmatrix}$	$\begin{Bmatrix}0.8175 & 1\\ 0.7970 & 0.2030\end{Bmatrix}$
β	$\beta_0=3,\beta_1=15$	$\beta_0=4,\beta_1=1$	$\beta_0=4,\beta_1=2$	$\beta_0=4,\beta_1=3$
z **ω**	$\begin{Bmatrix}0.8304 & 0.9999\\ 0.7829 & 0.2121\end{Bmatrix}$	$\begin{Bmatrix}1 & 0\\ 0.3001 & 0.6999\end{Bmatrix}$	$\begin{Bmatrix}1 & 0\\ 0.2064 & 0.7936\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.1749\\ 0.1723 & 0.8277\end{Bmatrix}$
β	$\beta_0=4,\beta_1=4$	$\beta_0=4,\beta_1=5$	$\beta_0=4,\beta_1=6$	$\beta_0=4,\beta_1=7$
z **ω**	$\begin{Bmatrix}0.3708 & 1\\ 0.8235 & 0.1765\end{Bmatrix}$	$\begin{Bmatrix}0.4932 & 1\\ 0.8182 & 0.1818\end{Bmatrix}$	$\begin{Bmatrix}0.5763 & 1\\ 0.8136 & 0.1864\end{Bmatrix}$	$\begin{Bmatrix}0.6362 & 1\\ 0.8099 & 0.1991\end{Bmatrix}$
β	$\beta_0=4,\beta_1=8$	$\beta_0=4,\beta_1=9$	$\beta_0=4,\beta_1=10$	$\beta_0=4,\beta_1=11$
z **ω**	$\begin{Bmatrix}0.6813 & 1\\ 0.8069 & 0.1931\end{Bmatrix}$	$\begin{Bmatrix}0.7165 & 1\\ 0.8045 & 0.1955\end{Bmatrix}$	$\begin{Bmatrix}0.7447 & 1\\ 0.8024 & 0.1976\end{Bmatrix}$	$\begin{Bmatrix}0.7679 & 1\\ 0.8008 & 0.1992\end{Bmatrix}$
β	$\beta_0=4,\beta_1=12$	$\beta_0=4,\beta_1=13$	$\beta_0=4,\beta_1=14$	$\beta_0=4,\beta_1=15$
z **ω**	$\begin{Bmatrix}0.7871 & 1\\ 0.7992 & 0.2008\end{Bmatrix}$	$\begin{Bmatrix}0.8035 & 0.9999\\ 0.7981 & 0.2019\end{Bmatrix}$	$\begin{Bmatrix}0.8175 & 1\\ 0.7970 & 0.2030\end{Bmatrix}$	$\begin{Bmatrix}0.8304 & 0.9999\\ 0.7829 & 0.2121\end{Bmatrix}$
β	$\beta_0=5,\beta_1=1$	$\beta_0=5,\beta_1=2$	$\beta_0=5,\beta_1=3$	$\beta_0=5,\beta_1=4$
z **ω**	$\begin{Bmatrix}1 & 0\\ 0.3001 & 0.6999\end{Bmatrix}$	$\begin{Bmatrix}1 & 0\\ 0.2064 & 0.7936\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.1749\\ 0.1723 & 0.8277\end{Bmatrix}$	$\begin{Bmatrix}0.3708 & 1\\ 0.8235 & 0.1765\end{Bmatrix}$
β	$\beta_0=5,\beta_1=5$	$\beta_0=5,\beta_1=6$	$\beta_0=5,\beta_1=7$	$\beta_0=5,\beta_1=8$
z **ω**	$\begin{Bmatrix}0.4932 & 1\\ 0.8182 & 0.1818\end{Bmatrix}$	$\begin{Bmatrix}0.5763 & 1\\ 0.8136 & 0.1864\end{Bmatrix}$	$\begin{Bmatrix}0.6362 & 1\\ 0.8099 & 0.1991\end{Bmatrix}$	$\begin{Bmatrix}0.6813 & 1\\ 0.8069 & 0.1931\end{Bmatrix}$
β	$\beta_0=5,\beta_1=9$	$\beta_0=5,\beta_1=10$	$\beta_0=5,\beta_1=11$	$\beta_0=5,\beta_1=12$
z **ω**	$\begin{Bmatrix}0.7165 & 1\\ 0.8045 & 0.1955\end{Bmatrix}$	$\begin{Bmatrix}0.7447 & 1\\ 0.8024 & 0.1976\end{Bmatrix}$	$\begin{Bmatrix}0.7679 & 1\\ 0.8008 & 0.1992\end{Bmatrix}$	$\begin{Bmatrix}0.7871 & 1\\ 0.7992 & 0.2008\end{Bmatrix}$
β	$\beta_0=5,\beta_1=13$	$\beta_0=5,\beta_1=14$	$\beta_0=5,\beta_1=15$	$\beta_0=6,\beta_1=1$
z **ω**	$\begin{Bmatrix}0.8035 & 0.9999\\ 0.7981 & 0.2019\end{Bmatrix}$	$\begin{Bmatrix}0.8175 & 1\\ 0.7970 & 0.2030\end{Bmatrix}$	$\begin{Bmatrix}0.8304 & 0.9999\\ 0.7829 & 0.2121\end{Bmatrix}$	$\begin{Bmatrix}1 & 0\\ 0.3001 & 0.6999\end{Bmatrix}$
β	$\beta_0=6,\beta_1=2$	$\beta_0=6,\beta_1=3$	$\beta_0=6,\beta_1=4$	$\beta_0=6,\beta_1=5$
z **ω**	$\begin{Bmatrix}1 & 0\\ 0.2064 & 0.7936\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.1749\\ 0.1723 & 0.8277\end{Bmatrix}$	$\begin{Bmatrix}0.3708 & 1\\ 0.8235 & 0.1765\end{Bmatrix}$	$\begin{Bmatrix}0.4932 & 1\\ 0.8182 & 0.1818\end{Bmatrix}$
β	$\beta_0=6,\beta_1=6$	$\beta_0=6,\beta_1=7$	$\beta_0=6,\beta_1=8$	$\beta_0=6,\beta_1=9$
z **ω**	$\begin{Bmatrix}0.5763 & 1\\ 0.8136 & 0.1864\end{Bmatrix}$	$\begin{Bmatrix}0.6362 & 1\\ 0.8099 & 0.1991\end{Bmatrix}$	$\begin{Bmatrix}0.6813 & 1\\ 0.8069 & 0.1931\end{Bmatrix}$	$\begin{Bmatrix}0.7165 & 1\\ 0.8045 & 0.1955\end{Bmatrix}$
β	$\beta_0=6,\beta_1=10$	$\beta_0=6,\beta_1=11$	$\beta_0=6,\beta_1=12$	$\beta_0=6,\beta_1=13$

z **ω**	$\begin{Bmatrix}0.7447 & 1\\ 0.8025 & 0.1975\end{Bmatrix}$	$\begin{Bmatrix}0.7679 & 1\\ 0.8008 & 0.1992\end{Bmatrix}$	$\begin{Bmatrix}0.7871 & 1\\ 0.7992 & 0.2008\end{Bmatrix}$	$\begin{Bmatrix}0.8035 & 0.9999\\ 0.7981 & 0.2019\end{Bmatrix}$
β	$\beta_0=6,\beta_1=14$	$\beta_0=6,\beta_1=15$	$\beta_0=7,\beta_1=1$	$\beta_0=7,\beta_1=2$
z **ω**	$\begin{Bmatrix}0.8175 & 1\\ 0.7970 & 0.2030\end{Bmatrix}$	$\begin{Bmatrix}0.8304 & 0.9999\\ 0.7829 & 0.2121\end{Bmatrix}$	$\begin{Bmatrix}1 & 0\\ 0.3001 & 0.6999\end{Bmatrix}$	$\begin{Bmatrix}1 & 0\\ 0.2064 & 0.7936\end{Bmatrix}$
β	$\beta_0=7,\beta_1=3$	$\beta_0=7,\beta_1=4$	$\beta_0=7,\beta_1=5$	$\beta_0=7,\beta_1=6$
z **ω**	$\begin{Bmatrix}1 & 0.1749\\ 0.1723 & 0.8277\end{Bmatrix}$	$\begin{Bmatrix}0.3708 & 1\\ 0.8235 & 0.1765\end{Bmatrix}$	$\begin{Bmatrix}0.4932 & 1\\ 0.8182 & 0.1818\end{Bmatrix}$	$\begin{Bmatrix}0.5763 & 1\\ 0.8136 & 0.1864\end{Bmatrix}$
β	$\beta_0=7,\beta_1=7$	$\beta_0=7,\beta_1=8$	$\beta_0=7,\beta_1=9$	$\beta_0=7,\beta_1=10$
z **ω**	$\begin{Bmatrix}0.6362 & 1\\ 0.8099 & 0.1991\end{Bmatrix}$	$\begin{Bmatrix}0.6813 & 1\\ 0.8069 & 0.1931\end{Bmatrix}$	$\begin{Bmatrix}0.7165 & 1\\ 0.8045 & 0.1955\end{Bmatrix}$	$\begin{Bmatrix}0.7447 & 1\\ 0.8024 & 0.1976\end{Bmatrix}$
β	$\beta_0=7,\beta_1=11$	$\beta_0=7,\beta_1=12$	$\beta_0=7,\beta_1=13$	$\beta_0=7,\beta_1=14$
z **ω**	$\begin{Bmatrix}0.7679 & 1\\ 0.8008 & 0.1992\end{Bmatrix}$	$\begin{Bmatrix}0.7872 & 1\\ 0.7991 & 0.2009\end{Bmatrix}$	$\begin{Bmatrix}0.8035 & 0.9999\\ 0.7981 & 0.2019\end{Bmatrix}$	$\begin{Bmatrix}0.8176 & 1\\ 0.7963 & 0.2037\end{Bmatrix}$
β	$\beta_0=7,\beta_1=15$	$\beta_0=8,\beta_1=1$	$\beta_0=8,\beta_1=2$	$\beta_0=8,\beta_1=3$
z **ω**	$\begin{Bmatrix}0.8304 & 0.9999\\ 0.7829 & 0.2121\end{Bmatrix}$	$\begin{Bmatrix}1 & 0\\ 0.3001 & 0.6999\end{Bmatrix}$	$\begin{Bmatrix}1 & 0\\ 0.2064 & 0.7936\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.1749\\ 0.1723 & 0.8277\end{Bmatrix}$
β	$\beta_0=8,\beta_1=4$	$\beta_0=8,\beta_1=5$	$\beta_0=8,\beta_1=6$	$\beta_0=8,\beta_1=7$
z **ω**	$\begin{Bmatrix}0.3708 & 1\\ 0.8235 & 0.1765\end{Bmatrix}$	$\begin{Bmatrix}0.4932 & 1\\ 0.8182 & 0.1818\end{Bmatrix}$	$\begin{Bmatrix}0.5763 & 1\\ 0.8136 & 0.1864\end{Bmatrix}$	$\begin{Bmatrix}0.6362 & 1\\ 0.8099 & 0.1991\end{Bmatrix}$
β	$\beta_0=8,\beta_1=8$	$\beta_0=8,\beta_1=9$	$\beta_0=8,\beta_1=10$	$\beta_0=8,\beta_1=11$
z **ω**	$\begin{Bmatrix}0.6813 & 1\\ 0.8069 & 0.1931\end{Bmatrix}$	$\begin{Bmatrix}0.7165 & 1\\ 0.8045 & 0.1955\end{Bmatrix}$	$\begin{Bmatrix}0.7447 & 1\\ 0.8024 & 0.1976\end{Bmatrix}$	$\begin{Bmatrix}0.7679 & 1\\ 0.8008 & 0.1992\end{Bmatrix}$
β	$\beta_0=8,\beta_1=12$	$\beta_0=8,\beta_1=13$	$\beta_0=8,\beta_1=14$	$\beta_0=8,\beta_1=15$
z **ω**	$\begin{Bmatrix}0.7872 & 1\\ 0.7991 & 0.2009\end{Bmatrix}$	$\begin{Bmatrix}0.8035 & 0.9999\\ 0.7981 & 0.2019\end{Bmatrix}$	$\begin{Bmatrix}0.8176 & 1\\ 0.7963 & 0.2037\end{Bmatrix}$	$\begin{Bmatrix}0.8304 & 0.9999\\ 0.7829 & 0.2121\end{Bmatrix}$
β	$\beta_0=9,\beta_1=1$	$\beta_0=9,\beta_1=2$	$\beta_0=9,\beta_1=3$	$\beta_0=9,\beta_1=4$
z **ω**	$\begin{Bmatrix}1 & 0\\ 0.3001 & 0.6999\end{Bmatrix}$	$\begin{Bmatrix}1 & 0\\ 0.2064 & 0.7936\end{Bmatrix}$	$\begin{Bmatrix}1 & 0.1749\\ 0.1723 & 0.8277\end{Bmatrix}$	$\begin{Bmatrix}0.3708 & 1\\ 0.8235 & 0.1765\end{Bmatrix}$
β	$\beta_0=9,\beta_1=5$	$\beta_0=9,\beta_1=6$	$\beta_0=9,\beta_1=7$	$\beta_0=9,\beta_1=8$
z **ω**	$\begin{Bmatrix}0.4932 & 1\\ 0.8182 & 0.1818\end{Bmatrix}$	$\begin{Bmatrix}0.5763 & 1\\ 0.8136 & 0.1864\end{Bmatrix}$	$\begin{Bmatrix}0.6362 & 1\\ 0.8099 & 0.1991\end{Bmatrix}$	$\begin{Bmatrix}0.6813 & 1\\ 0.8069 & 0.1931\end{Bmatrix}$
β	$\beta_0=9,\beta_1=9$	$\beta_0=9,\beta_1=10$	$\beta_0=9,\beta_1=11$	$\beta_0=9,\beta_1=12$
z **ω**	$\begin{Bmatrix}0.7165 & 1\\ 0.8045 & 0.1955\end{Bmatrix}$	$\begin{Bmatrix}0.7447 & 1\\ 0.8024 & 0.1976\end{Bmatrix}$	$\begin{Bmatrix}0.7679 & 1\\ 0.8008 & 0.1992\end{Bmatrix}$	$\begin{Bmatrix}0.7872 & 1\\ 0.7991 & 0.2009\end{Bmatrix}$

β	$\beta_0=9,\beta_1=13$	$\beta_0=9,\beta_1=14$	$\beta_0=9,\beta_1=15$	$\beta_0=10,\beta_1=1$
z **ω**	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$	$\begin{Bmatrix} 0.8176 & 1 \\ 0.7963 & 0.2037 \end{Bmatrix}$	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.3001 & 0.6999 \end{Bmatrix}$
β	$\beta_0=10,\beta_1=2$	$\beta_0=10,\beta_1=3$	$\beta_0=10,\beta_1=4$	$\beta_0=10,\beta_1=5$
z **ω**	$\begin{Bmatrix} 1 & 0 \\ 0.2064 & 0.7936 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1749 \\ 0.1723 & 0.8277 \end{Bmatrix}$	$\begin{Bmatrix} 0.3708 & 1 \\ 0.8235 & 0.1765 \end{Bmatrix}$	$\begin{Bmatrix} 0.4932 & 1 \\ 0.8182 & 0.1818 \end{Bmatrix}$
β	$\beta_0=10,\beta_1=6$	$\beta_0=10,\beta_1=7$	$\beta_0=10,\beta_1=8$	$\beta_0=10,\beta_1=9$
z **ω**	$\begin{Bmatrix} 0.5763 & 1 \\ 0.8136 & 0.1864 \end{Bmatrix}$	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8099 & 0.1991 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.8069 & 0.1931 \end{Bmatrix}$	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8045 & 0.1955 \end{Bmatrix}$
β	$\beta_0=10,\beta_1=10$	$\beta_0=10,\beta_1=11$	$\beta_0=10,\beta_1=12$	$\beta_0=10,\beta_1=13$
z **ω**	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8024 & 0.1976 \end{Bmatrix}$	$\begin{Bmatrix} 0.7679 & 1 \\ 0.8008 & 0.1992 \end{Bmatrix}$	$\begin{Bmatrix} 0.7872 & 1 \\ 0.7991 & 0.2009 \end{Bmatrix}$	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$
β	$\beta_0=10,\beta_1=14$	$\beta_0=10,\beta_1=15$	$\beta_0=11,\beta_1=1$	$\beta_0=11,\beta_1=2$
z **ω**	$\begin{Bmatrix} 0.8176 & 1 \\ 0.7963 & 0.2037 \end{Bmatrix}$	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.3001 & 0.6999 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.2064 & 0.7936 \end{Bmatrix}$
β	$\beta_0=11,\beta_1=3$	$\beta_0=11,\beta_1=4$	$\beta_0=11,\beta_1=5$	$\beta_0=11,\beta_1=6$
z **ω**	$\begin{Bmatrix} 1 & 0.1749 \\ 0.1723 & 0.8277 \end{Bmatrix}$	$\begin{Bmatrix} 0.3708 & 1 \\ 0.8235 & 0.1765 \end{Bmatrix}$	$\begin{Bmatrix} 0.4932 & 1 \\ 0.8182 & 0.1818 \end{Bmatrix}$	$\begin{Bmatrix} 0.5763 & 1 \\ 0.8136 & 0.1864 \end{Bmatrix}$
β	$\beta_0=11,\beta_1=7$	$\beta_0=11,\beta_1=8$	$\beta_0=11,\beta_1=9$	$\beta_0=11,\beta_1=10$
z **ω**	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8099 & 0.1991 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.8069 & 0.1931 \end{Bmatrix}$	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8045 & 0.1955 \end{Bmatrix}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8024 & 0.1976 \end{Bmatrix}$
β	$\beta_0=11,\beta_1=11$	$\beta_0=11,\beta_1=12$	$\beta_0=11,\beta_1=13$	$\beta_0=11,\beta_1=14$
z **ω**	$\begin{Bmatrix} 0.7679 & 1 \\ 0.8008 & 0.1992 \end{Bmatrix}$	$\begin{Bmatrix} 0.7872 & 1 \\ 0.7991 & 0.2009 \end{Bmatrix}$	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$	$\begin{Bmatrix} 0.8176 & 1 \\ 0.7963 & 0.2037 \end{Bmatrix}$
β	$\beta_0=11,\beta_1=15$	$\beta_0=12,\beta_1=1$	$\beta_0=12,\beta_1=2$	$\beta_0=12,\beta_1=3$
z **ω**	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.3001 & 0.6999 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.2064 & 0.7936 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1749 \\ 0.1723 & 0.8277 \end{Bmatrix}$
β	$\beta_0=12,\beta_1=4$	$\beta_0=12,\beta_1=5$	$\beta_0=12,\beta_1=6$	$\beta_0=12,\beta_1=7$
z **ω**	$\begin{Bmatrix} 0.3708 & 1 \\ 0.8235 & 0.1765 \end{Bmatrix}$	$\begin{Bmatrix} 0.4932 & 1 \\ 0.8182 & 0.1818 \end{Bmatrix}$	$\begin{Bmatrix} 0.5763 & 1 \\ 0.8136 & 0.1864 \end{Bmatrix}$	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8099 & 0.1991 \end{Bmatrix}$
β	$\beta_0=12,\beta_1=8$	$\beta_0=12,\beta_1=9$	$\beta_0=12,\beta_1=10$	$\beta_0=12,\beta_1=11$
z **ω**	$\begin{Bmatrix} 0.6813 & 1 \\ 0.8069 & 0.1931 \end{Bmatrix}$	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8045 & 0.1955 \end{Bmatrix}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8024 & 0.1976 \end{Bmatrix}$	$\begin{Bmatrix} 0.7679 & 1 \\ 0.8008 & 0.1992 \end{Bmatrix}$
β	$\beta_0=12,\beta_1=12$	$\beta_0=12,\beta_1=13$	$\beta_0=12,\beta_1=14$	$\beta_0=12,\beta_1=15$

$\mathbf{z}$ ω	$\begin{Bmatrix} 0.7872 & 1 \\ 0.7991 & 0.2009 \end{Bmatrix}$	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$	$\begin{Bmatrix} 0.8176 & 1 \\ 0.7963 & 0.2037 \end{Bmatrix}$	$\begin{Bmatrix} 0.8186 & 0.9993 \\ 0.8162 & 0.1838 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=13,\beta_1=1$	$\beta_0=13,\beta_1=2$	$\beta_0=13,\beta_1=3$	$\beta_0=13,\beta_1=4$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0 \\ 0.3001 & 0.6999 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.2064 & 0.7936 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1749 \\ 0.1723 & 0.8277 \end{Bmatrix}$	$\begin{Bmatrix} 0.3708 & 1 \\ 0.8235 & 0.1765 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=13,\beta_1=5$	$\beta_0=13,\beta_1=6$	$\beta_0=13,\beta_1=7$	$\beta_0=13,\beta_1=8$
$\mathbf{z}$ ω	$\begin{Bmatrix} 0.4932 & 1 \\ 0.8182 & 0.1818 \end{Bmatrix}$	$\begin{Bmatrix} 0.5763 & 1 \\ 0.8136 & 0.1864 \end{Bmatrix}$	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8099 & 0.1991 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.8069 & 0.1931 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=13,\beta_1=9$	$\beta_0=13,\beta_1=10$	$\beta_0=13,\beta_1=11$	$\beta_0=13,\beta_1=12$
$\mathbf{z}$ ω	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8045 & 0.1955 \end{Bmatrix}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8024 & 0.1976 \end{Bmatrix}$	$\begin{Bmatrix} 0.7679 & 1 \\ 0.8008 & 0.1992 \end{Bmatrix}$	$\begin{Bmatrix} 0.7872 & 1 \\ 0.7991 & 0.2009 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=13,\beta_1=13$	$\beta_0=13,\beta_1=14$	$\beta_0=13,\beta_1=15$	$\beta_0=14,\beta_1=1$
$\mathbf{z}$ ω	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$	$\begin{Bmatrix} 0.8176 & 1 \\ 0.7963 & 0.2037 \end{Bmatrix}$	$\begin{Bmatrix} 0.8186 & 0.9993 \\ 0.8162 & 0.1838 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.3001 & 0.6999 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=14,\beta_1=2$	$\beta_0=14,\beta_1=3$	$\beta_0=14,\beta_1=4$	$\beta_0=14,\beta_1=5$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0 \\ 0.2064 & 0.7936 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1749 \\ 0.1723 & 0.8277 \end{Bmatrix}$	$\begin{Bmatrix} 0.3708 & 1 \\ 0.8235 & 0.1765 \end{Bmatrix}$	$\begin{Bmatrix} 0.4932 & 1 \\ 0.8182 & 0.1818 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=14,\beta_1=6$	$\beta_0=14,\beta_1=7$	$\beta_0=14,\beta_1=8$	$\beta_0=14,\beta_1=9$
$\mathbf{z}$ ω	$\begin{Bmatrix} 0.5763 & 1 \\ 0.8136 & 0.1864 \end{Bmatrix}$	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8099 & 0.1991 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.8069 & 0.1931 \end{Bmatrix}$	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8045 & 0.1955 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=14,\beta_1=10$	$\beta_0=14,\beta_1=11$	$\beta_0=14,\beta_1=12$	$\beta_0=14,\beta_1=13$
$\mathbf{z}$ ω	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8024 & 0.1976 \end{Bmatrix}$	$\begin{Bmatrix} 0.7679 & 1 \\ 0.8008 & 0.1992 \end{Bmatrix}$	$\begin{Bmatrix} 0.7872 & 1 \\ 0.7991 & 0.2009 \end{Bmatrix}$	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=14,\beta_1=14$	$\beta_0=14,\beta_1=15$	$\beta_0=15,\beta_1=1$	$\beta_0=15,\beta_1=2$
$\mathbf{z}$ ω	$\begin{Bmatrix} 0.8176 & 1 \\ 0.7963 & 0.2037 \end{Bmatrix}$	$\begin{Bmatrix} 0.8186 & 0.9993 \\ 0.8162 & 0.1838 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.3001 & 0.6999 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 \\ 0.2064 & 0.7936 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=15,\beta_1=3$	$\beta_0=15,\beta_1=4$	$\beta_0=15,\beta_1=5$	$\beta_0=15,\beta_1=6$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0.1749 \\ 0.1723 & 0.8277 \end{Bmatrix}$	$\begin{Bmatrix} 0.3708 & 1 \\ 0.8235 & 0.1765 \end{Bmatrix}$	$\begin{Bmatrix} 0.4932 & 1 \\ 0.8182 & 0.1818 \end{Bmatrix}$	$\begin{Bmatrix} 0.5763 & 1 \\ 0.8136 & 0.1864 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=15,\beta_1=7$	$\beta_0=15,\beta_1=8$	$\beta_0=15,\beta_1=9$	$\beta_0=15,\beta_1=10$
$\mathbf{z}$ ω	$\begin{Bmatrix} 0.6362 & 1 \\ 0.8099 & 0.1991 \end{Bmatrix}$	$\begin{Bmatrix} 0.6813 & 1 \\ 0.8069 & 0.1931 \end{Bmatrix}$	$\begin{Bmatrix} 0.7165 & 1 \\ 0.8045 & 0.1955 \end{Bmatrix}$	$\begin{Bmatrix} 0.7447 & 1 \\ 0.8024 & 0.1976 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0=15,\beta_1=11$	$\beta_0=15,\beta_1=12$	$\beta_0=15,\beta_1=13$	$\beta_0=15,\beta_1=14$
$\mathbf{z}$ ω	$\begin{Bmatrix} 0.7679 & 1 \\ 0.8008 & 0.1992 \end{Bmatrix}$	$\begin{Bmatrix} 0.7872 & 1 \\ 0.7991 & 0.2009 \end{Bmatrix}$	$\begin{Bmatrix} 0.8035 & 0.9999 \\ 0.7981 & 0.2019 \end{Bmatrix}$	$\begin{Bmatrix} 0.6937 & 0.9369 \\ 0.9126 & 0.0874 \end{Bmatrix}$

$\boldsymbol{\beta}$	$\beta_0 = 15, \beta_1 = 15$	-	-	-
$\mathbf{z}$ ω	$\begin{Bmatrix} 0.8186 & 0.9993 \\ 0.8162 & 0.1838 \end{Bmatrix}$	-	-	-

Three support points design: Table-3 provides locally A-optimal designs is for $\boldsymbol{\beta} = (\beta_0, \beta_1)^T$ where $\beta_0, \beta_1 \in [1,10]$ with region space [0,1].

Table-3

$\boldsymbol{\beta}$	$\beta_0 = 1, \beta_1 = 1$	$\beta_0 = 1, \beta_1 = 2$	$\beta_0 = 1, \beta_1 = 3$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1500 & 0.7000 & 0.1500 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1032 & 0.7936 & 0.1032 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1750 & 0.1750 \\ 0.1723 & 0.6554 & 0.1723 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 1, \beta_1 = 4$	$\beta_0 = 1, \beta_1 = 5$	$\beta_0 = 1, \beta_1 = 6$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0.3708 & 0.3708 \\ 0.1764 & 0.6472 & 0.1764 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.4932 & 0.4932 \\ 0.1817 & 0.6366 & 0.1817 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.5763 & 0.5763 \\ 0.1863 & 0.6274 & 0.1863 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 1, \beta_1 = 7$	$\beta_0 = 1, \beta_1 = 8$	$\beta_0 = 1, \beta_1 = 9$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0.6362 & 1 \\ 0.0950 & 0.8100 & 0.0950 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6813 & 1 \\ 0.0965 & 0.8070 & 0.0965 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7165 & 0.7165 \\ 0.1954 & 0.6092 & 0.1954 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 1, \beta_1 = 10$	$\beta_0 = 2, \beta_1 = 1$	$\beta_0 = 2, \beta_1 = 2$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0.7447 & 1 \\ 0.0987 & 0.8026 & 0.0987 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1500 & 0.7000 & 0.1500 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1032 & 0.7936 & 0.1032 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 2, \beta_1 = 3$	$\beta_0 = 2, \beta_1 = 4$	$\beta_0 = 2, \beta_1 = 5$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0.1750 & 0.1750 \\ 0.1723 & 0.6554 & 0.1723 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.3708 & 0.3708 \\ 0.1764 & 0.6472 & 0.1764 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.4932 & 0.4932 \\ 0.1817 & 0.6366 & 0.1817 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 2, \beta_1 = 6$	$\beta_0 = 2, \beta_1 = 7$	$\beta_0 = 2, \beta_1 = 8$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0.5763 & 0.5763 \\ 0.1863 & 0.6274 & 0.1863 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6362 & 1 \\ 0.0950 & 0.8100 & 0.0950 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6813 & 1 \\ 0.0965 & 0.8070 & 0.0965 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 2, \beta_1 = 9$	$\beta_0 = 2, \beta_1 = 10$	$\beta_0 = 3, \beta_1 = 1$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0.7165 & 0.7165 \\ 0.1954 & 0.6092 & 0.1954 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7447 & 1 \\ 0.0987 & 0.8026 & 0.0987 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1500 & 0.7000 & 0.1500 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 3, \beta_1 = 2$	$\beta_0 = 3, \beta_1 = 3$	$\beta_0 = 3, \beta_1 = 4$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1032 & 0.7936 & 0.1032 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1750 & 0.1750 \\ 0.1723 & 0.6554 & 0.1723 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.3708 & 0.3708 \\ 0.1764 & 0.6472 & 0.1764 \end{Bmatrix}$
$\boldsymbol{\beta}$	$\beta_0 = 3, \beta_1 = 5$	$\beta_0 = 3, \beta_1 = 6$	$\beta_0 = 3, \beta_1 = 7$
$\mathbf{z}$ ω	$\begin{Bmatrix} 1 & 0.4932 & 0.4932 \\ 0.1817 & 0.6366 & 0.1817 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.5763 & 0.5763 \\ 0.1863 & 0.6274 & 0.1863 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6362 & 1 \\ 0.0950 & 0.8100 & 0.0950 \end{Bmatrix}$

β	$\beta_0=3,\beta_1=8$	$\beta_0=3,\beta_1=9$	$\beta_0=3,\beta_1=10$
z **ω**	$\begin{Bmatrix} 1 & 0.6813 & 1 \\ 0.0965 & 0.8070 & 0.0965 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7165 & 0.7165 \\ 0.1954 & 0.6092 & 0.1954 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7447 & 1 \\ 0.0987 & 0.8026 & 0.0987 \end{Bmatrix}$
β	$\beta_0=4,\beta_1=1$	$\beta_0=4,\beta_1=2$	$\beta_0=4,\beta_1=3$
z **ω**	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1500 & 0.7000 & 0.1500 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1032 & 0.7936 & 0.1032 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1750 & 0.1750 \\ 0.1723 & 0.6554 & 0.1723 \end{Bmatrix}$
β	$\beta_0=4,\beta_1=4$	$\beta_0=4,\beta_1=5$	$\beta_0=4,\beta_1=6$
z **ω**	$\begin{Bmatrix} 1 & 0.3708 & 0.3708 \\ 0.1764 & 0.6472 & 0.1764 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.4932 & 0.4932 \\ 0.1817 & 0.6366 & 0.1817 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.5763 & 0.5763 \\ 0.1863 & 0.6274 & 0.1863 \end{Bmatrix}$
β	$\beta_0=4,\beta_1=7$	$\beta_0=4,\beta_1=8$	$\beta_0=4,\beta_1=9$
z **ω**	$\begin{Bmatrix} 1 & 0.6362 & 1 \\ 0.0950 & 0.8100 & 0.0950 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6813 & 1 \\ 0.0965 & 0.8070 & 0.0965 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7165 & 0.7165 \\ 0.1954 & 0.6092 & 0.1954 \end{Bmatrix}$
β	$\beta_0=4,\beta_1=10$	$\beta_0=5,\beta_1=1$	$\beta_0=5,\beta_1=2$
z **ω**	$\begin{Bmatrix} 1 & 0.7447 & 1 \\ 0.0987 & 0.8026 & 0.0987 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1500 & 0.7000 & 0.1500 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1032 & 0.7936 & 0.1032 \end{Bmatrix}$
β	$\beta_0=5,\beta_1=3$	$\beta_0=5,\beta_1=4$	$\beta_0=5,\beta_1=5$
z **ω**	$\begin{Bmatrix} 1 & 0.1750 & 0.1750 \\ 0.1723 & 0.6554 & 0.1723 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.3708 & 0.3708 \\ 0.1764 & 0.6472 & 0.1764 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.4932 & 0.4932 \\ 0.1817 & 0.6366 & 0.1817 \end{Bmatrix}$
β	$\beta_0=5,\beta_1=6$	$\beta_0=5,\beta_1=7$	$\beta_0=5,\beta_1=8$
z **ω**	$\begin{Bmatrix} 1 & 0.5763 & 0.5763 \\ 0.1863 & 0.6274 & 0.1863 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6362 & 1 \\ 0.0950 & 0.8100 & 0.0950 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6813 & 1 \\ 0.0965 & 0.8070 & 0.0965 \end{Bmatrix}$
β	$\beta_0=5,\beta_1=9$	$\beta_0=5,\beta_1=10$	$\beta_0=6,\beta_1=1$
z **ω**	$\begin{Bmatrix} 1 & 0.7165 & 0.7165 \\ 0.1954 & 0.6092 & 0.1954 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7447 & 1 \\ 0.0987 & 0.8026 & 0.0987 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1500 & 0.7000 & 0.1500 \end{Bmatrix}$
β	$\beta_0=6,\beta_1=2$	$\beta_0=6,\beta_1=3$	$\beta_0=6,\beta_1=4$
z **ω**	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1032 & 0.7936 & 0.1032 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1750 & 0.1750 \\ 0.1723 & 0.6554 & 0.1723 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.3708 & 0.3708 \\ 0.1764 & 0.6472 & 0.1764 \end{Bmatrix}$
β	$\beta_0=6,\beta_1=5$	$\beta_0=6,\beta_1=6$	$\beta_0=6,\beta_1=7$
z **ω**	$\begin{Bmatrix} 1 & 0.4932 & 0.4932 \\ 0.1817 & 0.6366 & 0.1817 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.5763 & 0.5763 \\ 0.1863 & 0.6274 & 0.1863 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6362 & 1 \\ 0.0950 & 0.8100 & 0.0950 \end{Bmatrix}$
β	$\beta_0=6,\beta_1=8$	$\beta_0=6,\beta_1=9$	$\beta_0=6,\beta_1=10$
z **ω**	$\begin{Bmatrix} 1 & 0.6813 & 1 \\ 0.0965 & 0.8070 & 0.0965 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7165 & 0.7165 \\ 0.1954 & 0.6092 & 0.1954 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7447 & 1 \\ 0.0987 & 0.8026 & 0.0987 \end{Bmatrix}$
β	$\beta_0=7,\beta_1=1$	$\beta_0=7,\beta_1=2$	$\beta_0=7,\beta_1=3$

z **ω**	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1500 & 0.7000 & 0.1500 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1032 & 0.7936 & 0.1032 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1750 & 0.1750 \\ 0.1723 & 0.6554 & 0.1723 \end{Bmatrix}$
β	$\beta_0 = 7, \beta_1 = 4$	$\beta_0 = 7, \beta_1 = 5$	$\beta_0 = 7, \beta_1 = 6$
z **ω**	$\begin{Bmatrix} 1 & 0.3708 & 0.3708 \\ 0.1764 & 0.6472 & 0.1764 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.4932 & 0.4932 \\ 0.1817 & 0.6366 & 0.1817 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.5763 & 0.5763 \\ 0.1863 & 0.6274 & 0.1863 \end{Bmatrix}$
β	$\beta_0 = 7, \beta_1 = 7$	$\beta_0 = 7, \beta_1 = 8$	$\beta_0 = 7, \beta_1 = 9$
z **ω**	$\begin{Bmatrix} 1 & 0.6362 & 1 \\ 0.0950 & 0.8100 & 0.0950 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6813 & 1 \\ 0.0965 & 0.8070 & 0.0965 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7165 & 0.7165 \\ 0.1954 & 0.6092 & 0.1954 \end{Bmatrix}$
β	$\beta_0 = 7, \beta_1 = 10$	$\beta_0 = 8, \beta_1 = 1$	$\beta_0 = 8, \beta_1 = 2$
z **ω**	$\begin{Bmatrix} 1 & 0.7447 & 1 \\ 0.0987 & 0.8026 & 0.0987 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1500 & 0.7000 & 0.1500 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1032 & 0.7936 & 0.1032 \end{Bmatrix}$
β	$\beta_0 = 8, \beta_1 = 3$	$\beta_0 = 8, \beta_1 = 4$	$\beta_0 = 8, \beta_1 = 5$
z **ω**	$\begin{Bmatrix} 1 & 0.1750 & 0.1750 \\ 0.1723 & 0.6554 & 0.1723 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.3708 & 0.3708 \\ 0.1764 & 0.6472 & 0.1764 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.4932 & 0.4932 \\ 0.1817 & 0.6366 & 0.1817 \end{Bmatrix}$
β	$\beta_0 = 8, \beta_1 = 6$	$\beta_0 = 8, \beta_1 = 7$	$\beta_0 = 8, \beta_1 = 8$
z **ω**	$\begin{Bmatrix} 1 & 0.5763 & 0.5763 \\ 0.1863 & 0.6274 & 0.1863 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6362 & 1 \\ 0.0950 & 0.8100 & 0.0950 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6813 & 1 \\ 0.0965 & 0.8070 & 0.0965 \end{Bmatrix}$
β	$\beta_0 = 8, \beta_1 = 9$	$\beta_0 = 8, \beta_1 = 10$	$\beta_0 = 9, \beta_1 = 1$
z **ω**	$\begin{Bmatrix} 1 & 0.7165 & 0.7165 \\ 0.1954 & 0.6092 & 0.1954 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7447 & 1 \\ 0.0987 & 0.8026 & 0.0987 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1500 & 0.7000 & 0.1500 \end{Bmatrix}$
β	$\beta_0 = 9, \beta_1 = 2$	$\beta_0 = 9, \beta_1 = 3$	$\beta_0 = 9, \beta_1 = 4$
z **ω**	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1032 & 0.7936 & 0.1032 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1750 & 0.1750 \\ 0.1723 & 0.6554 & 0.1723 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.3708 & 0.3708 \\ 0.1764 & 0.6472 & 0.1764 \end{Bmatrix}$
β	$\beta_0 = 9, \beta_1 = 5$	$\beta_0 = 9, \beta_1 = 6$	$\beta_0 = 9, \beta_1 = 7$
z **ω**	$\begin{Bmatrix} 1 & 0.4932 & 0.4932 \\ 0.1817 & 0.6366 & 0.1817 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.5763 & 0.5763 \\ 0.1863 & 0.6274 & 0.1863 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6362 & 1 \\ 0.0950 & 0.8100 & 0.0950 \end{Bmatrix}$
β	$\beta_0 = 9, \beta_1 = 8$	$\beta_0 = 9, \beta_1 = 9$	$\beta_0 = 9, \beta_1 = 10$
z **ω**	$\begin{Bmatrix} 1 & 0.6813 & 1 \\ 0.0965 & 0.8070 & 0.0965 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7165 & 0.7165 \\ 0.1954 & 0.6092 & 0.1954 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7447 & 1 \\ 0.0987 & 0.8026 & 0.0987 \end{Bmatrix}$
β	$\beta_0 = 10, \beta_1 = 1$	$\beta_0 = 10, \beta_1 = 2$	$\beta_0 = 10, \beta_1 = 3$
z **ω**	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1500 & 0.7000 & 0.1500 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0 & 1 \\ 0.1032 & 0.7936 & 0.1032 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.1750 & 0.1750 \\ 0.1723 & 0.6554 & 0.1723 \end{Bmatrix}$
β	$\beta_0 = 10, \beta_1 = 4$	$\beta_0 = 10, \beta_1 = 5$	$\beta_0 = 10, \beta_1 = 6$
z **ω**	$\begin{Bmatrix} 1 & 0.3708 & 0.3708 \\ 0.1764 & 0.6472 & 0.1764 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.4932 & 0.4932 \\ 0.1817 & 0.6366 & 0.1817 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.5763 & 0.5763 \\ 0.1863 & 0.6274 & 0.1863 \end{Bmatrix}$

β	$\beta_0 = 10, \beta_1 = 7$	$\beta_0 = 10, \beta_1 = 8$	$\beta_0 = 10, \beta_1 = 9$
z ω	$\begin{Bmatrix} 1 & 0.6362 & 1 \\ 0.0950 & 0.8100 & 0.0950 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.6813 & 1 \\ 0.0965 & 0.8070 & 0.0965 \end{Bmatrix}$	$\begin{Bmatrix} 1 & 0.7165 & 0.7165 \\ 0.1954 & 0.6092 & 0.1954 \end{Bmatrix}$
β	$\beta_0 = 10, \beta_1 = 10$	-	-
z ω	$\begin{Bmatrix} 1 & 0.7447 & 1 \\ 0.0987 & 0.8026 & 0.0987 \end{Bmatrix}$	-	-