Right Truncated Lomax Distribution: Properties and Estimation

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Abstract

In this paper, we propose the truncated Lomax distribution. Fundamental properties of the new distribution are studied such as; moments, moment generating and characteristic function, quantile function, incomplete moments, Lorenz and Bonferroni curves, order statistics and Rényi entropy. Maximum likelihood estimators are derived in case of complete sample, Type I and Type II censored samples. An approximate confidence interval of the parameters is obtained for large sample sizes. Simulation issue is executed in order to investigate the performance of estimators.

Keywords: Lomax distribution; Maximum likelihood method; Type I censoring and Type II censoring.

1. Introduction

A truncated distribution is defined as a conditional distribution that results from restricting the $f(t|a \le T < b)$ domain of the statistical distribution. Hence, truncated distributions are used in cases where occurrences are limited to values which lie above or below a given threshold or within a specified range.
Let T be a random variable from a distribution with a probability density function (pdf), say $f(t)$, cumulative distribution function (cdf), say $F(t)$, and the range of the support $(-\infty,\infty)$. The density function of T defined in $a < T < b$ is given by

$$f(t|a \le T < b) = \begin{cases} \dfrac{g(t)}{G(b)-G(a)} & a \le t < b \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

(see, Lawless (2003)). Because is scaled up to account for the probability of being in the restricted support, this function is a density function. The restriction can occur either on a single side of the range which is called singly truncated or on both sides of the range which is called doubly truncated. If occurrences are limited to values which lie below a given threshold, the lower (left) truncated distribution is obtained. Similarly, if occurrences are limited to values which lie above a given threshold, the upper (right) truncated distribution arises.

Several truncated distributions have been provided by many authors. Some of the recent truncated distributions are; truncated Weibull distribution (Kantar and Usta, 2015); doubly truncated Fréchet distribution (Abid, 2016); generalized exponential truncated negative binomial distribution (Jayakumar and Sankaran, 2017); truncated inverted generalized exponential distribution (Genç, 2017); right truncated normal distribution (Thomopoulos, 2018), truncated Weibull Frèchet distribution (Hassan *et al.*, 2019 a), and truncated power Lomax distribution (Hassan *et al.*, 2019 b) among others.

Lomax (1954) provided a heavy-tail probability distribution, the so called Lomax (L) distribution, often used in business, economics, and actuarial modeling. It is widely applied in many areas, for instance, analysis of income and wealth data, modeling business failure data, biological sciences, model firm size and queuing problems (see Harris, 1968; Atkinson and Harrison, 1978; and Hassan and Al-Ghamdi, 2009). The (cdf) and (pdf) of the Lomax distribution are given, respectively, by

$$F_L(t;\alpha,\lambda)=1-\lambda^{\alpha}(\lambda+t)^{-\alpha}, \qquad t>0, \tag{2}$$

and

$$f_L(t;\alpha,\lambda)=\alpha\lambda^{\alpha}(\lambda+t)^{-(\alpha+1)}, \tag{3}$$

where, $\alpha,\lambda>0$ are the shape and scale parameters(Ps) respectively.

In this article, we are motivated to define a new truncated distribution referred to right truncated L (RTL) distribution. We obtain some main properties of the new distribution and discuss maximum likelihood estimation of its Ps based on complete and censored samples. This article can be organized as follows. The pdf, cdf, and hazard rate function (hrf) of the RTL model are defined in Section 2. Section 3 provides some statistical properties of RTL distribution. The maximum likelihood estimators and simulation study are provided in Section 4. At the end, concluding remarks are outlined in Section 5.

2. Right Truncated Lomax Distribution

In this section, we introduce the information of the RTL distribution

Definition: *A random variable X is said to have the RTL distribution (or [0,1] RTL distribution) with shape Ps* α *and scale parameter* $\lambda=1$, *if its pdf is constructed by employing (1) for* $a=0, b=1$ *with cdf (2) and pdf (3) as follows*

$$f_{RTL}(x;\alpha)=\frac{g(x;\alpha,1)}{G(1;\alpha,1)-G(0;\alpha,1)}=\frac{\alpha(1+x)^{-(\alpha+1)}}{1-2^{-\alpha}}, \qquad 0<x<1. \tag{4}$$

A random variable X with distribution (4) is denoted by $X\sim \text{RTL}(\alpha)$. The cdf related to (4) is given by

$$F_{RTL}(x;\alpha)=\frac{G(x;\alpha,1)-G(0;\alpha,1)}{G(1;\alpha,1)-G(0;\alpha,1)}=\frac{1-(1+x)^{-\alpha}}{1-2^{-\alpha}}. \qquad (5)$$

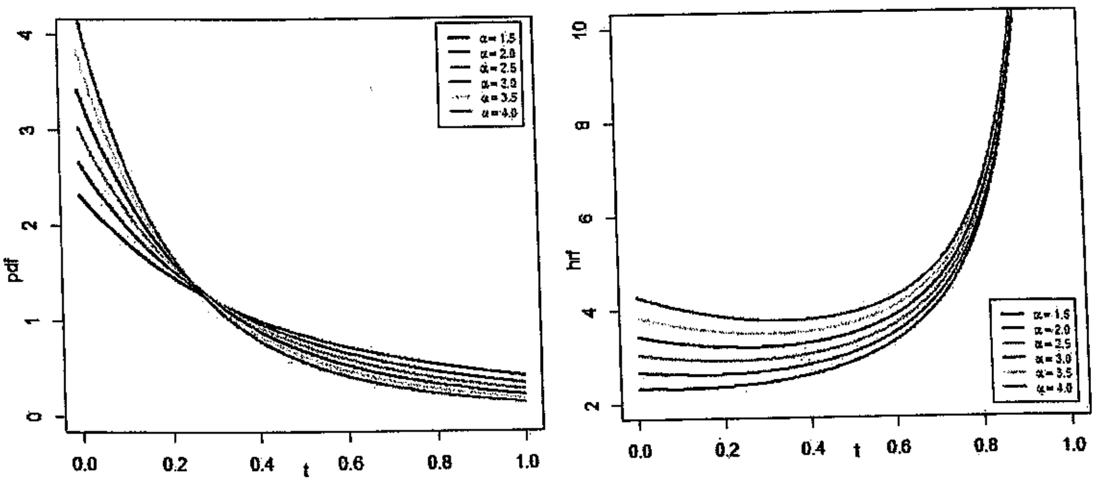
Depending on pdf (4) and cdf (5), the survival function and hazard rate function (hrf) are given by

$$\overline{F}_{RTL}(x;\alpha)=\frac{(1+x)^{-\alpha}-2^{-\alpha}}{1-2^{-\alpha}},$$

and,

$$h_{RTL}(x;\alpha)=\frac{\alpha(1+x)^{-(\alpha+1)}}{(1+x)^{-\alpha}-2^{-\alpha}}.$$

A variety of possible shapes of the pdf and the hrf of the TL distribution for some choices of values of parameter are represented in Figure 1.



(a) (b)

Figure 1. The pdf and hrf of RTL distribution for different values of parameters

It can be detected from Figure 1 that the pdf shape can be uni-model, reversed J-shaped and right skewed. Also, the shape of the hrf of the TL distribution could be increasing, and J -shaped.

In addition, the reversed hrf and cumulative hrf are obtained respectively as follows

$$\tau_{RTL}(x;\alpha)=\frac{\alpha(1+x)^{-(\alpha+1)}}{1-(1+x)^{-\alpha}},$$

and,

$$H_{RTL}(x;\alpha)=-ln\left(\frac{(1+x)^{-\alpha}-2^{-\alpha}}{1-2^{-\alpha}}\right).$$

The quantile function denoted by $Q(u)$ of random variable X has a cdf (5) is obtained as follows:

$$Q(u)=\left[1-u(1-2^{-\alpha})\right]^{\frac{-1}{\alpha}}-1, \tag{6}$$

where, the random variable u belongs to the uniform distribution on $[0,1]$. The 1st, 2nd, and 3rd quartiles are obtained from (6) by taking and $u=0.25, 0.5$ and 0.75 respectively.

3. Basic Properties

In this section we give some statistical properties of RTL distribution.

3.1 Moments

Since the moments are necessary and important in any statistical analysis. So, we derive the r^{th} moment of the RTL distribution. If X has the pdf (4), then $\acute{\mu}_r$ is obtained as follows:

$$\mu'_r=\int_0^1 x^r f_{RTL}(x;\alpha)dx=\frac{\alpha}{1-2^{-\alpha}}\int_0^1 x^r(1+x)^{-(\alpha+1)}dx. \tag{7}$$

We employ the following binomial expansion:

$$(1+Z)^{-\theta}=\sum_{j=0}^{\infty}(-1)^j\binom{\theta+j-1}{j}Z^j, \tag{8}$$

for $(1+x)^{-(\alpha+1)}$ as follows:

$$(1+x)^{-(\alpha+1)} = \sum_{j=0}^{\infty} (-1)^j \binom{\alpha+j}{j} x^j. \qquad (9)$$

Substituting (9) in (7) then,

$$\mu'_r = A^* \int_0^1 x^{r+j} dx = \frac{A^*}{r+j+1}, \qquad (10)$$

where, $A^* = \frac{\alpha}{1-2^{-\alpha}} \sum_{j=0}^{\infty} (-1)^j \binom{\alpha+j}{j}$.

Furthermore, the moment generating function (mgf) and characteristic function (chf) of the RTL distribution respectively are derived as follows

$$M_x(t) = E(e^{tx}) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \mu'_r = \sum_{r=0}^{\infty} \frac{t^r}{r!} \frac{A^*}{r+j+1},$$

and,

$$\phi_x(t) = E(e^{itx}) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \frac{A^*}{r+j+1}.$$

3.2Incomplete moments and Inequality measures

The incomplete moments are used in many statistical areas especially in the income distribution for measuring inequality like; income quintiles, the Lorenz curve, Pietra and Gini measures of inequality. The s^{th} incomplete moment of the RTPL distribution is obtained as follows

$$\Phi_s(t) = \int_0^t x^s f_{RTL}(x;\alpha) dx = A^* \int_0^t x^{s+j} dx,$$

then,

$$\Phi_s(t) = A^* \frac{t^{s+j+1}}{s+j+1}.$$

Therefore, inequality measures are often calculated for distributions other than expenditure. The Lorenz $[L_F(z)]$ and Bonferroni $[B_F(z)]$ curves witch where, $$w_i = (-1)^i \binom{\delta(\alpha+1)+i-1}{i} \frac{\alpha^\delta}{(1-2^{-\alpha})^\delta}.$$

3.4Order statistics

Let $X_{(1)} < X_{(2)} < ... < X_{(n)}$ be the order statistics of a random sample of size n then, the pdf of the k^{th} order statistic is given by:

$$f_{x_{(k)}}(x) = \frac{f(x)}{B(k,n-k+1)} \sum_{v=0}^{n-k} (-1)^v \binom{n-k}{v} F(x)^{v+k-1} \tag{14}$$

where, $B(.,.)$ is the beta function. The pdf of the k^{th} order statistic of RTL distribution is obtained by substituting (4) and (5) in (14) as follows

$$f_{RTL_{x_{(k)}}}(x;\alpha) = \sum_{v=0}^{n-k} \frac{\alpha(-1)^v \binom{n-k}{v}}{(1-2^{-\alpha})^{v+k} B(k,n-k+1)} (1+x)^{-\alpha-1} (1-(1+x)^{-\alpha})^{v+k-1}.$$

Employing the following binomial expansion

$$(1-Z)^\theta = \sum_{u=0}^{\theta} (-1)^u \binom{\theta}{u} z^u,$$

for $(1-(1+x)^{-\alpha})^{v+k-1}$ as follows

$$(1-(1+x)^{-\alpha})^{v+k-1} = \sum_{u=0}^{v+k-1} (-1)^u \binom{v+k-1}{u} (1+x)^{-\alpha u}$$

$$f_{RTL_{x_{(k)}}}(x;\alpha) = \sum_{v=0}^{n-k} \sum_{u=0}^{v+k-1} \frac{\alpha(-1)^{v+u} \binom{n-k}{v} \binom{v+k-1}{u}}{(1-2^{-\alpha})^{v+k} B(k,n-k+1)} (1+x)^{-\alpha(u+1)-1}$$

By using binomial expansion in (8), we employ

$$f_{RTL_{x_{(k)}}}(x;\alpha) = \sum_{v=0}^{n-k} \sum_{u=0}^{v+k-1} \sum_{m=0}^{\infty} \frac{\alpha(-1)^{v+u+m} \binom{n-k}{v} \binom{v+k-1}{u} \binom{\alpha(u+1)+m}{m}}{(1-2^{-\alpha})^{v+k} B(k,n-k+1)} (x)^m.$$

Hence, the pdf of the k^{th} order statistic of RTL distribution is given by

$$f_{RTL_{x_{(k)}}}(x;\alpha) = \sum_{m=0}^{\infty} w_m x^m,$$

where,

$$w_m = \sum_{v=0}^{n-k}\sum_{u=0}^{v+k-1} \frac{\alpha(-1)^{v+u+m}\binom{n-k}{v}\binom{v+k-1}{u}\binom{\alpha(u+1)+m}{m}}{(1-2^{-\alpha})^{v+k} B(k,n-k+1)}.$$

Further, the r^{th} moment of k^{th} order statistics for RTL distribution is given by

$$E(X^r_{(k)}) = \sum_{m=0}^{\infty} w_m \int_0^1 x^{r+m} dx$$

$$= \sum_{m=0}^{\infty} \frac{w_m}{r+m+1}.$$

4. Estimation and Simulation Study

In this section, maximum likelihood (ML) estimators of the model parameters are derived in case of complete, Type I and Type II censored samples. Approximate confidence intervals are also obtained. Further, numerical study is provided.

4.1 Parameter Estimation Based on Censoring Samples

In reliability or lifetime testing experiments, most of the encountered data are censored due to various reasons such as time limitation, cost or other resources. Here, we discuss estimation of population Ps of the RTL distribution based on two censoring schemes; namely, Type I and Type II. In Type-I censoring (TIC), we have a fixed time say; ω, but the number of items fail during the experiment is random. Whereas, in Type-II censoring (TIIC) scheme, the experiment is continued (i.e. time varies) until the specified number of failures c occurs.

4.1.1 ML estimators in case of TIC

Suppose that $X_{(1)} < X_{(2)} < ... < X_{(n)}$ be a TIC sample of size n whose life time's follow RTL distribution (4) are placed on a life test and the test is terminated at determined time ω before all n items have failed. The number of failures c and all failure times are random variables. The likelihood function, based on TIC for RTL model is given by

$$L_1 = \frac{n!}{(n-c)!}\left[1-\frac{1-S_\omega^{-\alpha}}{1-2^{-\alpha}}\right]^{n-c}\left\{\prod_{i=1}^{c}\left[\frac{\alpha S_i^{-(\alpha+1)}}{1-2^{-\alpha}}\right]\right\},$$

where, $S_i = (1+x_i)$, and $S_\omega = (1+\omega)$, also for simplicity we write S_i instead of $S_{(i)}$.

Then, the log-likelihood function is obtained as follows

$$LnL_1 = \ln\left[\frac{n!}{(n-c)!}\right] + (n-c)\ln\left[1-\frac{1-S_\omega^{-\alpha}}{1-2^{-\alpha}}\right] - c\ln(1-2^{-\alpha}) + c\ln\alpha - (\alpha+1)\sum_{i=1}^{c}\ln S_i .$$

Then, the first partial derivative of the log-likelihood with respect to the unknown Ps is given by

$$\frac{d\ln L_1}{d\alpha} = \frac{c}{\alpha} - \frac{2^{-\alpha}c\ln 2}{1-2^{-\alpha}} + \frac{(n-c)\left(-\frac{S_\omega^{-\alpha}\ln S_\omega}{1-2^{-\alpha}} + \frac{2^{-\alpha}\ln 2(1-S_\omega^{-\alpha})}{(1-2^{-\alpha})^2}\right)}{1-\frac{1-S_\omega^{-\alpha}}{1-2^{-\alpha}}} - \sum_{i=1}^{c}\ln S_i .$$

Equating this partial derivative with zero and solving simultaneously yield the ML estimator of α based on TIC samples.

4.1.2 ML estimators in case of TIIC

If we want to ensure that the resulting data set contains a fixed number c of observed lifetimes and furthermore we want to terminate the test as fast as possible. So, the design must allow for the test to terminate at the c^{th} failure such that $X_{(1)} < X_{(2)} < \dots < X_{(c)}$ be a TIIC sample of size n observed from lifetime testing experiment. The likelihood function of RTL model, based on TIIC, is given by

$$L_2 = \frac{n!}{(n-c)!}\left[1-\frac{1-S_c^{-\alpha}}{1-2^{-\alpha}}\right]^{n-c}\left\{\prod_{i=1}^{c}\left[\frac{\alpha S_i^{-(\alpha+1)}}{1-2^{-\alpha}}\right]\right\}.$$

where, $S_i = (1+x_i)$, and $S_c = (1+c)$ also for simplicity we write S_i instead of $S_{(i)}$ then, the log-likelihood function, based on TIIC, is given by

$$LnL_2 = \ln\left[\frac{n!}{(n-c)!}\right] + (n-c)\ln\left[1-\frac{1-S_c^{-\alpha}}{1-2^{-\alpha}}\right] - c\ln(1-2^{-\alpha}) + c\ln\alpha - (\alpha+1)\sum_{i=1}^{c}\ln S_i .$$

Then, the first partial derivative of the log-likelihood are given by

$$\frac{d\ln L_2}{d\alpha} = \frac{c}{\alpha} - \frac{2^{-\alpha}c\ln 2}{1-2^{-\alpha}} + \frac{(n-c)\left(-\frac{S_c^{-\alpha}\ln S_c}{1-2^{-\alpha}} + \frac{2^{-\alpha}\ln 2(1-S_c^{-\alpha})}{(1-2^{-\alpha})^2}\right)}{1-\frac{1-S_c^{-\alpha}}{1-2^{-\alpha}}} - \sum_{i=1}^{c}\ln S_i .$$

Solving $\frac{d\ln L_2}{d\alpha} = 0$ numerically by using iteration technique, then the ML estimator of α is obtained via Mathematica 7.

Additionally, for $c = n$, we obtain the ML estimator under complete samples as seen in Tables 5-8.

For interval estimation of the Ps, it is known that under regularity condition, the asymptotic distribution of ML estimators of elements of unknown Ps for α is given by

$$(\hat{\alpha}-\alpha) \to N(0, I^{-1}(\alpha)),$$

where, $I^{-1}(\alpha)$ is the variance covariance matrix of unknown Ps α The elements of Fisher information matrix are obtained for both censoring schemes. Therefore, the two-sided approximate $\gamma 100$ percent limits for the ML estimates of a population Ps for α can be obtained, as follows:

$$L_{\alpha} = \hat{\alpha} - z_{\frac{\gamma}{2}}\sqrt{\operatorname{var}(\hat{\alpha})},\ U_{\alpha} = \hat{\alpha} + z_{\frac{\gamma}{2}}\sqrt{\operatorname{var}(\hat{\alpha})},$$

where z is the $100(1-\gamma/2)\%$ th standard normal percentile and var(.)'s denote the diagonal elements of variance covariance matrix corresponding to the model Ps.

4.2 Simulation Studies

Here, we provide numerical study to evaluate the behavior of the ML estimates of the RTL based on complete sample, TIC and TIIC schemes. The algorithm used here is outlined as follows:

- 1000 random sample of sizes $n = 30, 50$ and 100 are generated from the RTL distribution under TIC and TIIC.
- Exact values of Ps are chosen as; $\alpha = 0.7, 0.9, 1.5$ and 2.5.
- Three termination times are selected as $\omega = 0.7, \omega = 0.9$ and $\omega = 1$ based on TIC and the number of failure items; c, based on TIIC are selected as 70%, 90% and 100% (complete sample).
- The following measures are calculated

  i. Average ML of the simulated estimates $\hat{\alpha}_i, i = 1,2,...N$, where $N = 1000$

  $$\frac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i$$

  ii. Average bias of the simulated estimates $\hat{\alpha}_i, i = 1,2,...N$

  $$\frac{1}{N}\sum_{i=1}^{N}(\hat{\alpha}_i - \alpha).$$

iii. Average mean square error (MSE) of the simulated estimates $\hat{\alpha}_i, i=1,2,...N$

$$\frac{1}{N}\sum_{i=1}^{N}(\hat{\alpha}_i-\alpha)^2.$$

iv. Average length of the N simulated confidence intervals and coverage probability with confidence level $\gamma=90\%$ and 95% for $\hat{\alpha}_i, i=1,2,...N$ are calculated.

- Numerical results are recorded in Tables 1, 2, 3,... and 8.

Table 1: ML estimates, Biases, MSE, Average length and Coverage probability of RTL distribution under TIC for α= 0.7

n	ω	ML	Bias	MSE	Average length		Coverage probability	
					90%	95%	90%	95%
30	0.7	0.68956	-0.01044	0.89256	3.06796	3.65544	90.50	94.60
	0.9	0.69320	-0.00680	0.88792	3.05156	3.63590	90.10	94.90
	1	0.69332	-0.00668	0.88724	3.05103	3.63527	90.30	94.70
50	0.7	0.72788	0.02787	0.54091	2.36777	2.82117	89.30	94.00
	0.9	0.72757	0.02757	0.53554	2.35531	2.80633	89.30	94.60
	1	0.72752	0.02752	0.53430	2.35489	2.80582	89.30	94.50
100	0.7	0.68429	-0.01571	0.26091	1.66761	1.98694	90.70	95.10
	0.9	0.68612	-0.01389	0.25426	1.65878	1.97642	90.80	95.20
	1	0.68623	-0.01377	0.25412	1.65852	1.97610	90.60	95.20

Table 2: ML estimates, Biases, MSE, Average length and Coverage probability of RTL distribution under TIC for α= 0.9

n	ω	ML	Bias	MSE	Average length		Coverage probability	
					90%	95%	90%	95%
30	0.7	0.92065	0.02065	0.90903	3.08059	3.67050	90.50	94.50
	0.9	0.92709	0.02709	0.90163	3.06554	3.65256	90.50	94.50
	1	0.92765	0.02765	0.90054	3.06508	3.65201	90.40	94.70
50	0.7	0.89296	-0.00704	0.55493	2.37521	2.83004	89.40	95.00
	0.9	0.89514	-0.00486	0.55210	2.36333	2.81588	89.70	94.70
	1	0.89530	-0.00470	0.55187	2.36296	2.81544	89.70	94.80
100	0.7	0.90718	0.00717	0.27201	1.67430	1.99491	89.60	94.50
	0.9	0.90467	0.00467	0.26735	1.66588	1.98487	89.90	94.30
	1	0.90453	0.00453	0.26675	1.66560	1.98455	89.70	94.60

Table 3: ML estimates, Biases, MSE, Average length and Coverage probability of RTL distribution under TIC for α= 1.5

n	ω	ML	Bias	MSE	Average length		Coverage probability	
					90%	95%	90%	95%
30	0.7	1.52681	0.02681	0.90315	3.13147	3.73111	90.80	96.00
	0.9	1.52659	0.02659	0.89480	3.11776	3.71478	90.90	95.70
	1	1.52634	0.02634	0.89337	3.11728	3.71421	90.80	95.70
50	0.7	1.53042	0.03042	0.56505	2.41654	2.87928	89.00	94.80
	0.9	1.53219	0.03219	0.55705	2.40624	2.86700	89.50	95.20
	1	1.53166	0.03166	0.55695	2.40589	2.86659	89.50	95.10
100	0.7	1.48933	-0.01067	0.25590	1.70038	2.02598	90.80	96.30
	0.9	1.49232	-0.00768	0.25389	1.69325	2.01749	90.80	96.40
	1	1.49241	-0.00759	0.25386	1.69304	2.01724	90.60	96.40

Table 4: ML estimates, Biases, MSE, Average length and Coverage probability of RTL distribution under TIC for α= 2.5

n	ω	ML	Bias	MSE	Average length		Coverage probability	
					90%	95%	90%	95%
30	0.7	2.52467	0.02467	0.98968	3.27512	3.90227	91.10	95.90
	0.9	2.52726	0.02726	0.98375	3.26460	3.88973	90.50	95.80
	1	2.52709	0.02709	0.98262	3.26423	3.88929	90.60	95.70
50	0.7	2.56655	0.06655	0.62182	2.53274	3.01774	90.20	94.60
	0.9	2.56530	0.06530	0.61909	2.52431	3.00769	90.20	94.90
	1	2.56515	0.06515	0.61907	2.52406	3.00739	90.30	94.90
100	0.7	2.51467	0.01467	0.31188	1.78016	2.12104	89.10	94.40
	0.9	2.51533	0.01533	0.31028	1.77433	2.11409	89.30	94.30
	1	2.51531	0.01531	0.30986	1.77415	2.11389	89.30	94.30

Table 5: ML estimates, Biases, MSE, Average length and Coverage probability of RTL distribution under TIIC for α= 0.7

n	x_c	ML	Bias	MSE	Average length		Coverage probability	
					90%	95%	90%	95%
30	70%	0.73730	0.03730	0.95738	3.13165	3.73133	89.80	95.20
	90%	0.72034	0.02034	0.91137	3.05820	3.64381	90.10	95.30
	100%	0.71588	0.01588	0.90440	3.05278	3.63735	90.50	95.30
50	70%	0.70248	0.00248	0.58703	2.40858	2.86979	88.70	94.60
	90%	0.68772	-0.01228	0.55498	2.35681	2.80812	88.80	94.60
	100%	0.68579	-0.01422	0.55222	2.35375	2.80447	89.00	94.50
100	70%	0.71623	0.01623	0.24198	1.69386	2.01821	90.70	96.70
	90%	0.71143	0.01143	0.22932	1.66038	1.97833	91.50	96.50
	100%	0.71119	0.01119	0.22864	1.65870	1.97633	91.70	96.50

Table 6: ML estimates, Biases, MSE, Average length and Coverage probability of RTL distribution under TIIC for α= 0.9

n	x_c	ML	Bias	MSE	Average length		Coverage probability	
					90%	95%	90%	95%
30	70%	0.91347	0.01347	1.01390	3.15057	3.75387	89.40	94.80
	90%	0.89901	-0.00099	0.93671	3.07016	3.65806	88.80	95.20
	100%	0.89593	-0.00407	0.92817	3.06403	3.65076	88.80	95.00
50	70%	0.94048	0.04048	0.55752	2.42465	2.88895	89.80	96.20
	90%	0.93610	0.03610	0.52492	2.36777	2.82118	90.10	95.50
	100%	0.93488	0.03488	0.52356	2.36416	2.81687	90.30	95.30
100	70%	0.91980	0.01980	0.30361	1.70556	2.03215	88.50	93.20
	90%	0.91073	0.01073	0.29132	1.66822	1.98766	88.10	93.30
	100%	0.91082	0.01082	0.29090	1.66630	1.98538	88.20	93.20

Table 7: ML estimates, Biases, MSE, Average length and Coverage probability of RTL distribution under TIIC for α= 1.5

n	x_c	ML	Bias	MSE	Average length		Coverage probability	
					90%	95%	90%	95%
30	70%	1.52682	0.02682	1.00526	3.23055	3.84916	89.60	95.20
	90%	1.50604	0.00604	0.91806	3.12432	3.72260	89.60	95.10
	100%	1.50193	0.00193	0.90391	3.11505	3.71155	89.70	95.00
50	70%	1.55677	0.05677	0.61338	2.48997	2.96678	89.60	94.70
	90%	1.54111	0.04111	0.56068	2.41205	2.87393	90.90	95.00
	100%	1.54119	0.04119	0.55399	2.40661	2.86745	90.80	95.10
100	70%	1.52335	0.02335	0.27733	1.74772	2.08239	91.10	94.80
	90%	1.51580	0.01580	0.25723	1.69744	2.02249	90.50	95.10
	100%	1.51381	0.01381	0.25469	1.69432	2.01876	90.80	94.80

Table 8: ML estimates, Biases, MSE, Average length and Coverage probability of RTL distribution under TIIC for α= 2.5

n	x_c	ML	Bias	MSE	Average length		Coverage probability	
					90%	95%	90%	95%
30	70%	2.52723	0.02723	1.12656	3.44250	4.10170	90.40	95.70
	90%	2.50465	0.00465	1.00251	3.27711	3.90464	90.40	95.50
	100%	2.50401	0.00401	0.99109	3.26053	3.88489	90.70	95.70
50	70%	2.56260	0.06260	0.76494	2.65612	3.16474	88.30	93.40
	90%	2.54010	0.04010	0.68577	2.53302	3.01806	88.90	93.50
	100%	2.53098	0.03098	0.66971	2.52072	3.00341	89.00	93.80
100	70%	2.53804	0.03804	0.30333	1.86151	2.21797	91.40	96.20
	90%	2.52891	0.02891	0.27135	1.78086	2.12187	91.10	96.60
	100%	2.53033	0.03033	0.27003	1.77487	2.11474	91.30	96.60

From Tables 1-8 we conclude that

- As the sample size n increases the MSE of ML estimates decrease.
- As the termination time ω increases, the MSE of estimates decreases.
- Based on Tables 1-4, we can see that as the sample size n increases the MSE of estimates decreases and also that as the censoring level time ω increases, the MSE of estimates decreases .
- Based on Tables 1 to 8, the coverage probability is very close to the intended significance level for all values of n and α .
- The average length of confidence intervals for the unknown Ps decrease as n increases.

5. Summary and Conclusion

In this paper, we introduce a new model called the truncated Lomax. We investigate several structural properties of the new distribution, expressions for the ordinary moments, generating function and order statistics. The model parameters are estimated by the maximum likelihood method in case of complete and censored samples. A simulation study reveals that the estimates of the model have desirable properties such as, (i) the maximum likelihood estimates are not too far from the true parameter values; (ii) the biases and mean square errors of estimates in case of complete sample are smaller than the corresponding in censored samples; and (iii) the biases and the mean square values decrease as the sample size increases.

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