Resampling Techniques for Estimating the parameters of Grubbs model with asymmetric heavy-tailed distributions

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Abstract

In this paper, three resampling techniques are considered, namely, bootstrap, jackknife and jackknife after bootstrap. The main objective is to study the performance of these techniques for maximum likelihood estimation for the parameters using expectation conditional maximization either (ECME) algorithm for Grubbs model when the latent response follows asymmetric heavy-tailed distributions such as scale mixture of skew normal distributions (such as skew-t (ST), skew slash (SSL), skew contaminated normal (SCN)). Also, the performance of these techniques is discussed for detection of the influential observations using local influence method for assessing the robustness of these parameter estimates under different perturbation schemes for Grubbs model. The performance is illustrated through an application using real data set under different bootstrap replications. Our results provide resampling techniques with better fit, protect against outlying observations and more precise inferences than traditional techniques.

Keywords: Expectation conditional maximization either , Grubbs model, Jackknife after bootstrap, Scale mixture of skew normal distribution.

1 Introduction

The problem of comparing measurement devices which vary in price, time spent to measure and other features, such as efficiency, has been of growing interest in many scientific applications,such as medicine, Barnett[1969]. Grubbs measurement error model was introduced by Grubbs [1948]. This model is typically used to assess the relative agreement between two or more measuring devices (or instruments) that are used to measure the same quantity of interest. Lachos et al. [2007]studied the normal Grubbs (N-G) model, noting the well-known lack of robustness of least-square estimates against outlying observations. To overcome this deficiency, a general class of scale mixture of normal Grubbs model (SMN-G) was proposed inOsorio et al. [2009]. Properties of the SMN distributions, such as student-t (T), slash normal (SL) and contaminated normal (CN) may found in Andrews and Mallows [1974], Lange and Sinsheimer [1993]. For asymmetric setting, Montenegro et al. [2010] proposed the skew normal Grubbs (SN-G) model and showed advantages of using asymmetric distributions for obtaining accurate robust estimates. An asymmetric version of SMN distributions called scale mixture of skew normal distributions (SMSN) was introduced by da Silva Ferreira et al. [2011] as a challenging family for statistical procedures which accommodates both asymmetry and heavy tails jointly.This new family contains all the distributions which studied by Lange and Sinsheimer [1993] with an extra parameter, which regulates the skewness of the distribution such as skew-t (ST), skew slash (SSL) and skew contaminated Normal (SCN) distribution. Zeller et al. [2014] studied SMSN for Grubbs model and revealed to the results are perfect agreement with those presented in Osorio et al. [2009] and Montenegro et al. [2010] concluded that in the presence of outlying observations in measurement error models, supposing both asymmetry and heavy-tails, may represent a good choice for robust estimation.

Three resampling techniques will be considered in this paper. Bootstrap was proposed by Efron [1979], jackknife was developed by Efron [1982] and jackknife after bootstrap (JaB) was developed by Efron [1992]. Bootstrap and jackknife resampling techniques were considered by [Efron and Tibshirani [1993], Sahinler and Topuz [2007]] and they used the bootstrap and jackknife estimates in linear regression model. Also, both resampling were used by Algamal and Rasheed [2010] to estimate the sampling distribution of the parameter estimates in linear regression model. JaB was used by Martin and Roberts [2010] to determine the cut-off values for various diagnostic measures in linear regression under non-normal errors and small samples. JaB was used in Algamal [2012] in count regression model to assess the error in the bootstrap estimate parameters. Also, JaB was used in Beyaztas and Alin [2014c] to detect influential observations for binary logistic regression model. Diagnostic analysis is an efficient way to detect influential observations, this can be achieved by conducting local influence analysis under different perturbation schemes (Case-weight, Response perturbation, Multiplicative bias perturbation). Zhu and Lee [2001] developed an approach to perform local influence analysis for general statistical models with missing data by working with a Q-displacement function closely related to the conditional expectation of the complete-data log-likelihood at the E-step of the expectaion maximization (EM) algorithm. Zeller et al. [2014] used Zhu and Lee approach to detect the influential observations for SMSN-G models.

The primary objective of this paper is to evaluate the performances of resampling techniques such as bootstrap, jackknife and JaB in the estimation of the parameters for SMSN models, respectively, in addition, to detect the influential observations using the local influence method under different perturbation schemes following Zeller et al. [2014]. This paper is organized as follows: Section 2 describes the Grubbs model. Sections 3-5 discuss the performance of the resampling techniques in the estimation of the parameters, in the detection of the outlying observations and in the detection of the influential observations, respectively. Section 6 evaluates the performance of the resampling techniques through application using real data. Section 7 contains conclusions.

2 Grubbs Model

Suppose one have $p \geq 2$ instruments for measuring quantity of interest x in a group of n experimental units. Let x_i be the unobserved (true) value corresponding to unit i, where i represents the ith item in sample, y_i the measured value obtained in unit

$i, i = 1,...,n, j = 1,...,p$. Relating these variables, the Grubbs model is given by Grubbs [1973]

$$y_i = \alpha + 1_p x_i + \varepsilon_i \tag{2.1}$$

where $\alpha = (\alpha_1, \alpha_2, ..., \alpha_p)^T, \alpha_1 = 0$ to eliminate redundancy, $1_p = (1, ..., 1)^T$ are scalar $p \times 1$ vector, $y_i = (y_{i1}, ..., y_{ip})^T$ and $\varepsilon_i = (\varepsilon_{i1}, ...\varepsilon_{ip})^T$ (the error vector) are $p \times 1$ random vectors independent with $\varepsilon_i \overset{iid}{\sim} N_p(0, D(\zeta))$ and $x_i \overset{iid}{\sim} N_1(\mu_x, \zeta_x)$, where

$$D(\zeta) = diag(\zeta_1, ..., \zeta_p)^T. \tag{2.2}$$

$$Y \sim N(\mu, \Sigma), \; where \, \mu = \alpha + 1_p \mu_x, \; \Sigma = 1_p^T 1_p \zeta_x + D(\zeta) \tag{2.3}$$

Here the first instrument is compared to the remaining $p-1$ instruments.The parameter vector of interest is $\theta = (\mu_x, \alpha^T, \zeta_x, \zeta^T)$, in addition to λ_x in the skewness case, Montenegro et al. [2010].

3 Resampling Algorithmic Approach for Estimation of Parameters for Grubbs Model

This section is devoted to investigate the performance of resampling techniques such as bootstrap, jackknife and JaB in the estimation of the parameters for Grubbs models. Bootstrap and jackknife estimates of the parameters and the relevant standard error (SE) for Grubbs model can be obtained following the bootstrap and the jackknife algorithms described in Sahinler and Topuz [2007]. But the estimation of the parameters is to be conducted using expectation conditional maximization either(ECME) algorithm Liu and Rubin [1994] for SMSN models (ST, SSL, SCN) following Zeller et al. [2014], jackknife after bootstrap (JaB) estimates of the parameters and the relevant standard error for each estimate are obtained in Section 5 of Martin and Roberts [2010].

4 Resampling Algorithmic Approach for Detection Outlying Observations for Grubbs Model

To study the performance of resampling techniques in the detection of the outlying observations, The Mahalanobis distance was use by Zeller et al. [2014] as adiagnostic measure to detect the outlying observations for SMSN-G models,

$$d = (Y - \hat{\mu})^T \hat{\Sigma}^{-1} (Y - \hat{\mu}) \tag{4.1}$$

Jackknife and JaB techniques are considered in this section. With respect to the jackknife technique, one can detect the outlying observations, by removing the ith data point from the original data set, and compute the Mahalanobis distance for (n −1) observations, then a point is flagged as outlier if its distance value exceeds the cut-off value. The cut-off depends on the distribution of the Mahalanobis distance for ST, SSL and SCN distributions, Zeller et al. [2014] . After determining the outliers for each jackknife resample, calculate the percentage of data sets among all n reduced data sets in which each data point is flagged. This overall percentage will typically be a good indicator for this point to be an outlier, Martin et al. [2010]. For JaB technique, we can find the appropriate JaB influence cut-offs for a Mahalanobis distance to detect the outlying observations for Grubbs model following the algorithm in Martin and Roberts [2010].

5 Resampling Algorithmic Approach for Detection the Influential Observations for Grubbs Model

Diagnostic analysis is an efficient way to detect influential observations, this can be achieved by conducting local influence analysis to assess the influence of minor perturbations on the model/data under different perturbations schemes on the model/data such as (Case-weight, Response perturbation, Multiplicative bias perturbation). Zhu and Lee [2001] developed an approach to perform local influence analysis for general statistical models with missing data by working with a Q-displacement function closely related to the EM algorithm. Zeller et al. [2014] used Zhu and Lee

[2001] approach in the detection of the influential observations for SMSN-G models. Following Martin et al. [2010], we can develope and conduct the algorithm to evaluate the performance of jackknife in the detection of the influential observations for each distribution as follows:

1. Remove the ith data point from the original data set and compute the conformal normal curvature, Poon and Poon [1999] for (n −1) observations

$$B_{f_Q,l_j}(\theta) = C_{f_Q,l_j}(\theta)/(norm\left(2\Delta\left\{-\ddot{Q}_\theta\right\}^{-1}\Delta\right)), \tag{5.1}$$

where $C_{f_Q,l} = -2\left(l^T \ddot{Q}_{w_0} l\right)$, $-\ddot{Q}_{w_0} = \left(\Delta^T_{w_0}(-\ddot{Q}_\theta)^{-1}\Delta_{w_0}\right)$, l_j is a vector in R^p with the j^{th}entry equal to one and all other entries zero.

$$Q(\hat{\theta},\omega \mid \hat{\theta}) = E[l_c(\hat{\theta},\omega \mid Y_c) \mid y, \hat{\theta}],$$

$$\ddot{Q}_\theta(\theta) = \frac{\partial^2 Q(\theta|\hat{\theta})}{\partial\theta\partial\theta^T}, \; \Delta = \frac{\partial^2 Q(\theta,\omega|\hat{\theta})}{\partial\theta\partial\omega^T},$$

2. Determine if $M(0)$, $\left(M(0) = B_{f_Q,l}\right)$ exceeds the cut-off value $\overline{M(0)} + c^* SM(0)$, where c^* is a selected constant depending on the real application, $SM(0)$ is the standard deviation of $M(0)_j, j = 1, ..., q$,

3. Calculate the percentage of data sets among all n reduced data sets in which each observation is flagged, This overall percentage will typically be a good indicator for this observation to be an influential.

For JaB technique, one can follow Beyaztas and Alin [2013] algorithm to find an appropriate influence cut-offs for a conformal curvature normal measure for detection the influential observations for Grubbs models.

6 Application

A real example is presented to investigte the performance of resampling techniques in the estimation of the parameters and detection of the outlying and influential observations for Grubbs Model. Two instruments (a standard and a new one) used for measuring the vital capacity of the human lung, operated by skilled and unskilled operators, and administrated on a common group of 72 patients.The following four cases were compared: case 1: standard instrument and skilled operator; case 2: standard instrument and unskilled operator; case 3: New instrument and skilled operator; case 4: New instrument and unskilled operator, Barnett [1969]. The analysis conducted using R version 3.3.1. The number of replications (B) equal to 100, 1000. The convergence criterion

$$max_{j=1,\ldots,n_p} \mid \frac{\hat{\theta}^{(k+1)} - \hat{\theta}^{(k)}}{\hat{\theta}^{(k)}} \mid \leq \delta, \tag{6.1}$$

where n_p is the dimention of θ, δ is very small number, say 10^{-6} is used for the ECME algorithm. ML estimates and Jackknife estimates and the relevant standard error of these estimates for ST-G, SSL-G, SCN-G models are obtained. One can concluded that the jackknife estimates and the original estimates for each distribution are very close. Also, standard error of the jackknife estimates for ST, SSL, SCN distributions are less than SE of the estimates for the same distribution for original sample.

Bootstrap and JaB estimates and the relevant standard error of these estimates for ST-G, SSL-G,SCN-G models are obtained corresponding the number of replications B = 100, 1000. Noting that JaB estimates and bootstrap estimates are very close for each replication for SMSN-G models. SE of the Bootstrap and JaB estimates μ_x, ζ_x when B = 1000 are less than SE of the same estimates when B = 100 for SMSN-G models. Also, when the number of replication increases, one may got on good estimats with low standard error. But the standard error values for other estimates are not comparable because they are in different scales.

The relative efficiencies $RE1 = SE(\hat{\theta}_{boot})/SE(\hat{\theta}_{Jackk})$, $RE2 = SE(\hat{\theta}_{JaB})/SE(\hat{\theta}_{Jakk})$ are computed for each resampling technique for each model as shown in Table (1). One can conclude that the RE2 for JaB when B = 1000 is larger than the RE2 for JaB when B = 100 for all estimates.

Table 1: Relative Effeciency for bootstrap and JaB estimates for SMSN-G models when B=100,1000

		Relative Effieciency					
		ST-G		SCN-G		SSL-G	
B	Parameter	Bootstrap	JaB	Bootstrap	JaB	Bootstrap	JaB
100	μ_x	0.093	0.085	0.010	0.333	0.058	0.035
	ζ_x	0.125	0.089	0.095	0.155	3.249	1.086
	ζ_1	0.860	2.035	1.753	4.837	2.641	6.151
	ζ_2	1.047	8.855	1.027	10.934	2.007	19.226
	ζ_3	0.924	2.094	1.467	3.728	2.506	5.903
	ζ_4	1.063	2.640	1.212	3.135	2.465	6.258
	α_2	1.154	2.951	1.106	4.065	1.066	5.128
	α_3	1.017	1.526	1.052	1.683	0.901	2.403
	α_4	0.813	0.995	0.892	1.702	1.372	3.894
	λ_x	2.4	1.675	0.094	0.351	4.596	2.164
1000	μ_x	0.021	0.064	0.028	0.152	0.029	0.029
	ζ_x	0.043	0.035	0.008	0.053	0.880	0.880
	ζ_1	0.861	2.119	1.758	4.931	5.093	5.093
	ζ_2	1.007	9.047	0.982	11.733	17.742	17.742
	ζ_3	0.900	2.122	1.550	4.046	6.481	6.481
	ζ_4	1.057	2.699	1.235	3.297	6.780	6.780
	α_2	1.097	3.802	1.030	3.985	4.756	4.756
	α_3	0.99	1.541	1.035	1.743	2.054	2.054
	α_4	0.909	1.569	0.823	1.496	3.329	3.329
	λ_x	0.023	0.672	0.015	0.152	2.025	2.025

Bias estimate and the absolute relative bias are obtained for jackknife, bootstrap and JaB as shown in Table (2), where the absolute relative bias is the absolute value of the difference between jackknife, bootstrap or JaB estimator and the original estimator divided by original estimator, noting that the bias and the relative bias values for the bootstrap and JaB estimates less than bias and relative bias values for jackknife estimates. Also, bias and relative bias values for bootstrap and JaB estimates when B=1000 less than when B=100, mean's that using resampling technique when the number of replication increases help us to obtain good estimator with low bias or relative bias.

Table 2: **Bias estimate and the relative efficiency for all models for each resampling technique, when B = 100, 1000**

		Bias			Relative Bias		
B	Parameter	SCN	ST	SSL	SCN	ST	SSL
Jackknife	μ_x	2.698	0.852	-1.349	0.219	0.069	0.112
	ζ_x	-24.992	16.188	18.673	0.328	0.129	0.174
	ζ_1	0	1.207	-0.213	0	0.396	0.071
	ζ_2	0	0.284	-0.142	0	0.399	0.155
	ζ_3	0	0.852	-0.568	0	0.269	0.155
	ζ_4	0	0.852	-0.923	0	0.248	0.262
	α_2	0	0	0.071	0	0	0.129
	α_3	0	0	0.284	0	0	0.225
	α_4	0	-0.142	0.355	0	0.112	0.232
	λ_x	1.349	3.976	9.088	0.271	0.785	1.562
100	μ_x	0.604	1.523	0.4	0.049	0.124	0.033
	ζ_x	59	-3.592	15.753	0.773	0.0287	0.147
	ζ_1	2.387	0.688	0.836	1.172	0.225	0.281
	ζ_2	0.577	0.217	0.097	1.123	0.305	0.106
	ζ_3	1.963	0.421	0.855	1.062	0.133	0.235
	ζ_4	2.27	0.48	0.99	1.166	0.139	0.281
	α_2	-0.11	-0.038	-0.007	0.177	0.059	0.013
	α_3	-0.015	0.021	0.017	0.016	0.021	0.013
	α_4	-0.222	-0.04	-0.049	0.188	0.032	0.032
	λ_x	2.1	-0.06	2.029	0.422	0.012	0.349
1000	μ_x	0.948	1.329	0.04	0.077	0.108	0.003
	ζ_x	50.728	-0.423	15.253	0.665	0.003	0.142
	ζ_1	2.126	0.932	0.036	1.044	0.305	0.012
	ζ_2	0.512	0.303	0.069	0.996	0.426	0.075
	ζ_3	1.734	0.367	0.235	0.938	0.116	0.064
	ζ_4	1.918	0.472	0.505	0.985	0.138	0.144
	α_2	-0.073	-0.007	-0.067	0.117	0.011	0.122
	α_3	-0.046	0.047	-0.303	0.052	0.047	0.239
	α_4	-0.193	-0.037	-0.142	0.163	0.029	0.093
	λ_x	-0.816	0.256	1.244	0.164	0.051	0.214

The information criterion values are obtained for each resampling technique , also for each replication and for each distribution, following Montenegro et al. [2010], Zeller et al. [2014].Where the best distribution or the best technique is the one which have the smallest value of the information criterion. By looking at the values for the information criterion in Table(3), noting that the values of the information criterion values for boostrap and JaB techniques decreases when the number of replication increases for each distribution. Also, one can note that values of the information criterion for jackknife technique less than bootstrap and JaB at different replications.Values of the information criterion for all resampling techniques are less than values of the information criterion for the original sample. Indicating that, resampling techniques outperform the corresponding original sample for each distribution, indicates that these resampling techniques provides a better fit than the original sample. One can say, jackknife technnique is better based on the values of the information criterion for each distribution. But for comparing between distributions, one can note SCN-G is the better distribution. Values of information criterion for JaB not shown here because it is identical with bootstrap values at different replications.

Table 3: The information criteria values for SMSN-G models for resampling technique and each replication

Dist	B	Resampling methods	loglike	AIC	BIC	HQ
ST-G		Original Sample	-733.1815	744.1815	764.3278	752.2549
		Jackknife	-722.949	733.949	754.096	742.023
	100	Bootstrap	-729.757	740.757	760.904	748.831
	1000	Bootstrap	-729.228	739.228	757.543	746.568
SCN-G		Original Sample	-717.801	729.801	751.778	738.608
		Jackknife	-707.769	716.769	733.252	723.375
	100	Bootstrap	-719.314	731.314	753.292	740.122
	1000	Bootstrap	-718.310	725.311	735.705	729.534
SSL-G		Original sample	-735.941	746.941	767.087	755.014
		Jackknife	-725.681	736.681	756.827	744.755
	100	Bootstrap	-728.395	739.395	759.541	747.469
	1000	Bootstrap	-726.442	735.442	751.925	742.048

In order to detect outlying observations for Barnett data, the Mahalanobis distance can be used as a diagnostic measure to detect outluing observations, using equation(4.1). The outlying observations from original sample, jackknife and JaB when B=100 are detected and presented in Table(4) with cutoff values adopting the cutoff lines correspond to the quantile ($\xi = 0.95$), noting that the observations 7, 36, 49, 72 is the popular outlying observations for original sample, jackknife and JaB. The main conclusion is JaB technique flagged fewer outlying observations than original sample and jackknife technique for SCN-G models, but for ST-G, SSL-G JaB flagged the same obsevations for the original sample.

Table 4: **The outlying obersvations for SMSN-G models for original sample, jackknife and JaB when B=100**

Resampling technique	Dist	Cuttoff	Outliers
Original Sample	ST	13.036	7, 36, 49, 62
	SSL	12.375	7, 36, 49, 62
	SCN	9.488	1, 5, 10, 20, 27, 35, 36,37, 44,45, 47,48,49,52,54, 59, 62,71,72
Jackknife	ST	13.036	7, 36, 48, 61
	SSL	17.792	7, 48, 61
	SCN	9.488	1,5,7, 10, 20,27, 35,36,43,44,46, 47,48, 51,58,61 70,71
JaB	SCN	14.083	1,7,27,36,45, 47,49,52,62,72
	ST	14.801	7,36,49,62
	SSL	10.899	7,36,49,62

Table 5: **The influential obersvations which flagged for SMSN-G models under case weight and joint perturbation schemes for jackknife and JaB(B=100)**

	Perturbaton scheme	Case Weight		Joint perturbation		
Dist	Resampling technique	Cuttoff	Influential observa-tions	Dist	Cuttoff	Influential observations
SCN-G	Original sample	0.057	19, 24, 38, 45, 56, 59	SCN-G	1.177	-
	Jackknife	0.058	19,23,37,44, 55,58		1.229	-
	JaB	0.066	45, 56, 59		1.313	-
ST-G	Original sample	0.059	19, 24, 38, 44, 56, 59	ST-G	1.208	-
	Jackknife	0.057	19, 23, 37, 43,55, 58		1.222	-
	JaB	0.066	38, 44,56, 59		1.231	-
SSL-G	Original sample	0.039	44, 45, 59	SSL-G	1.236	-
	Jackknife	0.039	43, 44, 58		1.234	-
	JaB	0.044	-		1.245	-

Table 6: The influential obersvations which flagged for SMSN-G models under particular instrrument perturbation and multiplicative bias perturbation schemes for jackknife and JaB (B=100)

	Perturbaton scheme	Particular instrument perturbation		Multiplicative Bias		
Dist	Resampling technique	Cuttoff	Influential observations	Dist	Cuttoff	Influential observations
SCN-G	Original sample	0.032	-	SCN-G	1.393	-
	Jackknife	0.033	-		1.451	-
	JaB	0.052	-		1.533	-
ST-G	Original sample	0.035	14, 28, 32, 58	ST-G	1.433	-
	Jackknife	0.042	13, 27, 31, 57		1.442	-
	JaB	0.052	28		1.522	-
SSL-G	Original sample	0.032	2, 53	SSL-G	1.464	-
	Jackknife	0.032	2, 52		1.466	-
	JaB	0.041	-		1.511	-

The values of the conformal normal curvature B_i using equation(5.1) for original sample, jackknife and JaB are obtained to identify the influential observations for Barnett data set using the local influence approach under different perturbtion schemes. One can note from Table (5) that case weigh and under measurement perturbation for a particular instrument perturbation schems, JaB flagged fewer influential observations than jackknife and original sample for ST-G, SCN-G, SSL-G models.There is no influential observations for ST-G, SSL-G, SCN-G models under joint response and multiplicative bias perturbation schemes. as seen in Tables (5, 6).

7 Conclusion

The performance of resampling techniques, namely, bootstrap, jackknife, JaB are studied in the estimation of the parameters, detection of the outlying observations and detection of the influential observations for SMSN-G models.The main conclusion is that the use of these resampling techniques offer better fits, obtain ccurate estimates with low bias and low relative bias, protect against outliers and influentials observations than original sample especially when the number of replication increased. The nature of the JaB method needs much computation especially for larger sample sizes.To handle this problem and also to get more accurate results which can be implemented using sufficient bootstrap that was proposed by Singh and Sedory [2011] or one can implement sufficient JaB as in Beyaztas and Alin[2014a] to reduce the computing time which becomes very important for large samples for Grubbs model. In this paper, a single case deletion approach for jackknife and JaB. So, the detection of the influential points is a difficult problem, especially when there are masking and/or swamping effects which make it difficult to flag actual influential data points.To overcome such problems, one can implement (d-jackknife), Martin et al. [2010] and (d-JaB), Beyaztas and Alin [2014b] for Grubbs model.

References

1. Algamal, Z. Y. & Rasheed, K. B. (2010). Resampling in Linear Regression Model Using Jackknife and Bootstrap. *Iraqi Journal of Statistical Science*, 18, 59-73.

2. Algamal, Z.Y. and Rasheed, K.B. (2012). Resampling Techniques in Count data Regression Models. *Iraqi Journal of Statistical Science*, 15-5.

3. Andrews, D. F. and Mallows, C. L. (1974). Scale Mixtures of Normal Distributions. *Journal of the Royal Statistical Society, Series B (Methodological)*, 99-102.

4. Barnett, V. D. (1969). Simultaneous pairwise linear structural relationships. *Biometrics*, 129-142.

5. Beyaztas, U. and Alin, A. (2013). Jackknife After Bootstrap Method For Detection of Influential Observations in Linear Regression models. *Communications in Statistics-Simulation and Computation*, 42(6), 1256-1267.

6. Beyaztas, U. and Alin, A. (2014a). Sufficient Jackknife After Bootstrap Method for Detection of Influential Observations in Linear Regression Models. *Statistical Papers*, 55(4), 1001-1018.

7. Beyaztas, U. and Alin, A. (2014b). Delete-2 Jackknife After Bootstrap in Regression. *Quality and Reliability Engineering International*, 30(7), 993-1002.

8. Beyaztas, U. and Alin, A. (2014c). Jackknife After Bootstrap as Logistic Regression Diagnostic Tool. *Communications in Statistics-Simulation and Computation*, 43(9), 2047-2060.

9. Da Silva Ferreira, C. ,Bolfarine, H. & Lachos, V. H. (2011). Skew Scale Mixtures of Normal Distributions, Properties and Estimation. *Statistical Methodology* 8(2), 154-171.

10. Efron, B. (1979). Bootstrap methods: another look at the jackknife'Breakthroughs in Statistics'. Springer, 569-593.

11. Efron.B (1982).The Jackknife, The Bootstrap and Other Resampling Plans, *Society of Industrial and Applied Mathematics CBMS-NSF Monographs,* 38.

12. Efron, B. (1992). Jackknife After Bootstrap Standard Errors and Influence Functions. *Journal of the Royal Statistical Society, Series B (Methodological*), 83-127.

13. Efron, B and Tibshirani, R.J.(1993), *An introduction to the bootstrap*.Chapman and Hall.

14. Grubbs, F. E. (1948). On estimating precision of measuring instruments and product variability. *Journal of the American Statistical Association*, 43(242), 243-264.

15. Grubbs, F. E. (1973). Errors of Measurement, Precision, Accuracy and The Statistical Comparison of Measuring Instruments. *Technometrics*, 15(1), 53-66.

16. Lachos, V. H. ,Vilca, F. and Galea, M. (2007). Influence Diagnostics for the Grubbs's Model. *Statistical Papers*, 48(3), 419-436.

17. Lange, K. and Sinsheimer, J. S. (1993). Normal/Independent Distributions and their Applications in Robust Regression. *Journal of Computational and Graphical Statistics* 2(2), 175-198.

18. Liu, C. & Rubin, D. B. (1994). The ECME Algorithm: A simple Extension of EM and ECM with Faster Monotone Convergence. *Biometrika*, 633-648.

19. Martin, M. A. and Roberts, S. (2010). Jackknife After Bootstrap Regression Influence Diagnostics. *Journal of Nonparametric Statistics*, 22(2), 257-269.

20. Martin, M. A. , Roberts, S. and Zheng, L. (2010). Delete-2 and Delete-3 Jackknife Procedures for Unmasking in Regression. *Australian & New Zealand Journal of Statistics* 52(1).

21. Montenegro, L. C. , Lachos, V. H. and Bolfarine, H. (2010). Inference for a Skew Extension of the Grubbs Model. *Statistical Papers*, 51(3), 701-715.

22. Osorio, F. , Paula, G. A. and Galea, M. (2009). On Estimation and Influence Diagnostics for the Grubbs' Model Under Heavy-Tailed Distributions. *Computational Statistics & Data Analysis*, 53(4), 1249-1263.

23. Poon, W.Y. and Poon, Y. S. (1999). Conformal Normal Curvature and Assessment of Local Influence. *Journal of the Royal Statistical Society*: Series B (Statistical Methodology) 61(1), 51--61.

24. Sahinler, S. and Topuz, D. (2007). Bootstrap and Jackknife Resampling Algorithms for Estimation of Regression Parameters. *Journal of Applied Quantitative Methods*, 2(2), 188-199.

25. Singh, S. and Sedory, S. A. (2011). Sufficient Bootstrapping. *Computational statistics and data analysis*, 55(4), 1629-1637.

26. Zeller, C. B., Lachos, V. H. and Labra, F. V. (2014). Influence Diagnostics for Grubbs's Model with Asymmetric Heavy-Tailed Distributions. *Statistical Papers* 55(3), 671-690.

27. Zhu, H.-T. and Lee, S.-Y. (2001). Local Influence for Incomplete Data Models. *Journal of the Royal Statistical Society, Series B (Statistical Methodology)*, 63(1), 111-126.