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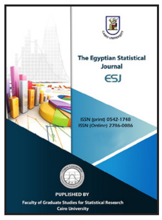

Parameter Estimation of an Agent-Based Model Under Loss Aversion

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A B S T R A C T

An agent-based model under loss aversion behavioral bias is introduced in Selim et al. (2015), however, without estimating its parameters. The proposed model proves great ability to replicate important stylized facts of real financial markets, such as random-walk prices, heavy-tailed returns distribution, clustered volatility, excess volatility, the absence of autocorrelation in raw returns, and the power-law autocorrelations in absolute returns, and fractal structure. However, the extent to which the model is able to predict the behavior of certain stock markets will be increased by estimating model parameters. In this article, the model parameters are estimated by conducting stability analysis and by indirect estimation. By this, policy makers can use this model as testbed to investigate the effect of any decision prior to applying it on the real stock market. Also, researchers can use this model to predict traders' behavior towards different hypotheses.

Keywords:

Agent-based modelling, parameter estimation, bifurcation analysis.

1. **Introduction**

Traditional economic models failed to capture the complex, high dynamic features of financial markets. This highlighted the significant role of agent-based approach in modeling financial markets. Many proposed Agent-based Modelling (ABM) [c.f. (Langton, 1986; Arifovic, 1996; Derveeuw, 2005; LeBaron & Tesfatsion, 2008; Bonabeau, 2002; Chen & Liao, 2005; Westerhoff, 2008; Feng et al., 2012; Lux, 2009; Farmer & Joshi, 2002) perform very well in replicating stylized facts of financial markets [c.f. (Mandelbrot, 1963; Fama, 1970; Cont, 2001)]. However, estimating model parameters remains a challenging process.

Many studies were run to estimate ABM parameters (Gilli & Winker, 2003; Alfarano et al., 2005; Alfarano et al., 2006; Alfarano et al., 2007; Manzan & Westerhoff, 2007; Amilon, 2008; Franke & Westerhoff, 2012; Boswijk et al., 2007; Winker & Gilli, 2001). Agent-based models could be used as testbeds for the decision-making process. This would reduce the risk of applying different rules and regulations directly on financial markets. Additionally, these agent-based models could be used by researchers to test different theories and hypotheses.

The main aim of this research is to estimate the parameters of the agent-based model proposed by Selim et al. (Selim et al., 2015). There are two agents in this model; (i) the market maker and (ii) traders. At each time step, traders decide on submitting an order or abstain from the market.

If the trader will participate, they can follow either technical (and becoming chartist traders) or fundamental (and becoming fundamentalist traders) trading strategy. The proposed model introduces loss-aversion behavioral bias to chartists to apply the prospect theory proposed by Tversky and Kahneman (Tversky & Kahneman, 1992; Tversky & Kahneman, 1991). The proposed model proves high ability to capture important stylized facts of financial markets, such as random-walk prices, volatility clustering, excess volatility heavy-tailed returns distribution and power-law tails. Thereafter, estimating model parameters is very important to enable using it by decision makers, investors, and researchers.

The following content is divided to four sections. In Section 2, the model under estimation is presented. Section 3 is devoted for stability and bifurcation analysis to estimate the parameters. In Section 4, simulation design and main results are presented. Finally, Section 5 concludes the paper.

2. The Model

A log-linear price impact function is used to describe behavior of the market maker (2002). The price settlement function values the relation between the quantity ordered (demand/supply) and the price of the asset. Therefore, the log-price of the asset in period t+1 is given by;

$$p_{t+1} = p_t + a\left(w_t^c D_t^c + w_t^f D_t^f\right) + \alpha_t \qquad (1)$$

where a is a positive price settlement parameter, D_t^c and D_t^f are the orders submitted by chartists and fundamentalists; respectively, at time t and w_t^c and w_t^f are the weights of technical strategy and fundamental strategy; respectively, at time t. Noise terms α_t are added to represent random factors affecting the price settlement process. $\alpha_t, t = 1, 2, \dots, T$ are assumed to be IID normally distributed random variables with mean zero and constant standard deviation σ_α.

Chartists follow technical analysis to exploit the price changes (Murphy, 1999). Orders utilizing technical trading rules at time t can be presented by;

$$D_t^c = b(p_t - p_{t-1}) + \beta_t \qquad (2)$$

where b is a positive reaction parameter that captures the strength of agents' sensitivity to price signals. In Eq. (2), the first term at the right-hand side of shows the difference between current and previous price, which represents the exploitation of price changes. The second term captures additional random orders of technical trading rules, where $\beta_t, t = 1, 2, \dots, T$ are IID normally distributed random variables with mean zero and constant standard deviation σ_β.

On the other hand, fundamental analysis assumes that prices will return to their fundamental values in the long run (Graham & Dodd, 2009). Orders created by fundamental trading rules at time t can be presented as;

$$D_t^f = c(F_t - p_t) + \gamma_t \qquad (3)$$

where c is a reaction parameter for the sensitivity of fundamentalists' excess demand to differences of the price from the underlying fundamental value. F_t are log-fundamental values (or simply fundamental values) (Day & Huang, 1990). γ_t is added to depict additional random orders of fundamental tradings. $\gamma_t, t = 1, 2, \dots, T$ are IID normally distributed random variables with mean zero and constant standard deviation σ_γ.

The evolutionary switching behavior between trading strategies, proposed by Brock and Hommes (1998), illustrates how agents' beliefs are evolved over time. The evolving behavior is mirrored in the weights $w_t = \{w_t^c, w_t^f, w_t^0\}$, where w_t^0 represents the fraction of inactive agents and w_t^c, w_t^f are as indicated in Eq. (1), thereafter, strategy weights add up to one. Weights are updated according to evolutionary fitness measure (or attractiveness of the trading rules) that could be presented as follows;

$$A_t^0 = 0 \qquad (4)$$

$$A_t^c = (\exp(p_t) - \exp(p_{t-1}))D_{t-2}^c + mA_{t-1}^c \qquad (5)$$

$$A_t^f = (\exp(p_t) - \exp(p_{t-1}))D_{t-2}^f + mA_{t-1}^f \qquad (6)$$

where A_t^c, A_t^f, and A_t^0 are the fitness measures of using chartist strategy, fundamental strategy, and no-trade strategy, respectively. Inactive traders submit zero orders, so they receive zero attractiveness of making such decision. Fitness measure of the other two agents; the chartists and the fundamentalists, relies on two components. The first term of the right-hand sides of (5) and (6) is the performance of the trading strategy in most recent time. Note that, orders submitted in period $t-2$ are implemented at the price stated in period $t-1$. The gains or losses depend on the price acknowledged in period t. The second term of the right-hand sides of (5) and (6) characterizes agents' memory, where $0 \leq m \leq 1$ is the memory parameter that evaluates the speed of recognizing present myopic profits. For $m = 0$, agent has no memory, while for $m = 1$ they calculate the fitness of the rule as a sum of all witnessed myopic profits.

Westerhoff (2008) proposed that agents symmetrically recognize gains and losses in terms of fitness. However, in this model a realistic behavioral bias is proposed, so that; chartists follow a value function of gains and losses to evaluate their strategies. The suggested value function indicates that, chartists recognize losses more than twice their perception of gains. We follow the Tversky and Kahneman (1991) and Benartzi and Thaler (1993) piecewise linear value function proposed by the prospect theory to apply the loss-aversion behavior. Consequently, the value of the fitness of technical strategy is provided by;

$$v_c = \begin{cases} A_t^c & if \quad A_t^c \geq 0 \\ \lambda\, A_t^c & if \quad A_t^c < 0 \end{cases} \qquad (7)$$

where λ is the parameter of loss aversion that measures the relative sensitivity to gains and losses. Nevertheless, setting $\lambda = 1$ reduces the value function to; $v_c = A_t^c$. This situation represents loss-neutral chartists.

The market share of each trading strategy could be measured by the discrete choice model[1] (Manski and McFadden (1981)), as follows;

$$w_t^c = \frac{\exp(rv_c)}{\exp(rA_t^0) + \exp(rv_c) + \exp(rA_t^f)} = \frac{\exp(rv_c)}{1 + \exp(rv_c) + \exp(rA_t^f)} \qquad (8)$$

$$w_t^f = \frac{\exp(rA_t^f)}{\exp(rA_t^0) + \exp(rv_c) + \exp(rA_t^f)} = \frac{\exp(rA_t^f)}{1 + \exp(rv_c) + \exp(rA_t^f)} \qquad (9)$$

$$w_t^0 = \frac{\exp(rA_t^0)}{\exp(rA_t^0) + \exp(rv_c) + \exp(rA_t^f)} = 1 - w_t^c - w_t^f \qquad (10)$$

The highest attractive strategy will be chosen by the more agents. The parameter r, in (8), (9), and (10) is called the intensity of choice and measures the sensitivity of mass of agents selecting the trading strategy with higher fitness measure. In such adaptive scheme, financial market prices and fractions of trading strategies will coevolve over time.

3. Stability and Bifurcation Analysis

In this section, we analyze the underlying deterministic system by dropping all the random terms and we characterize the unique steady state of the model; we also derive analytical conditions for the local asymptotic stability of the steady state and highlight their dependence on the key parameters of the model (i.e., the reaction coefficients of chartists and fundamentalists and the price adjustment coefficient). Though the evolution of the prices is due to high-dimensional non-

[1] A discrete choice model specifies probabilities $P(i|z,\theta)$ for each set of alternatives $\{i\}$ among which the decision maker can choose. The exogenous variables *z* describe observable attributes and characteristics of the decision maker and available alternatives to her/him. The parameters θ are to be estimated from the observed choices of a sample of decision makers. The choice probabilities are determined by the multinomial logit model as follows; $P(i|z,\theta) = \frac{\exp V_i(z,\theta)}{\sum_{j=1}^{M} \exp V_j(z,\theta)}$ where M is the number of available alternatives. And $V_i(z,\theta)$ is a summary statistic measuring the attractiveness of alternative *i*. It has the linear form of $V_i(z,\theta) = z_i, \theta, for\ i = 1,2,\dots,M$ CITATION Man81 \l 1033 | (Manski & McFadden, 1981)}.

linear laws of motion, it could be noticed that closed analysis is not prohibited due to the structure of the Jacobian of the deterministic system, evaluated at the steady state.

3.1 Deterministic Skeleton

Dropping the random terms from (1) – (3) and taking into account (4) – (10), we obtain a non-linear dynamic model with a high number of equations, some of which are second-order difference equations. However, the model can be reduced to a 5-D discrete-time dynamical system through suitable changes of variables as follows.

$$x_{t+1} = p_t$$

$$y_{t+1} = x_t \ (= p_{t-1})$$

The orders by technical traders at time *t* can thus be expressed as

$$D_t^c = b(p_t - x_t)$$

Rewriting one time ahead of (3.20) and (3.22) result in

$$A_{t+1}^f = (\exp(p_{t+1}) - \exp(p_t))D_{t-1}^f + mA_t^f,$$

And

$$A_{t+1}^c = (\exp(p_{t+1}) - \exp(p_t))D_{t-1}^c + mA_t^c$$

where D_{t-1}^c and D_{t-1}^f can be expressed by

$$D_{t-1}^c = b(p_{t-1} - x_{t-1}) = b(x_t - y_t)$$

$$D_{t-1}^f = c(f - x_t)$$

Therefore, we obtain the following 5-D system in the dynamic variables p, x, y, v^c and A^f .

$$p_{t+1} = p_t + a\left(w_t^c b(p_t - x_t) + w_t^f c(f_t - p_t)\right) \quad (11)$$

$$x_{t+1} = p_t \quad (12)$$

$$y_{t+1} = x_t \quad (13)$$

$$A_{t+1}^c = (\exp(p_{t+1}) - \exp(p_t))b(p_t - x_t) + mA_t^c \quad (14)$$

$$A_{t+1}^{f} = (\exp(p_{t+1}) - \exp(p_t))c(f - x_t) + mA_t^{f} \tag{15}$$

Notice that the dynamical model (11) – (15) is driven by the iteration of a 5-D map, which gives the state of the system at time *t*+1; described by $p_{t+1}, x_{t+1}, y_{t+1}, A_{t+1}^{c}$, and A_{t+1}^{f} as a function of the state of the system

at time *t*; i.e. p_t, x_t, y_t, A_t^{c}, and A_t^{f}. Note that, the other variables (w_t^{c} and w_t^{f}) and quantities $(\exp(p_{t+1}) - \exp(p_t))$ which appear in the right-hand sides of (14) and (15), respectively, are functions of the state at time *t* according to

$$w_t^{c} = \frac{\exp(rA_t^{c})}{\exp(rA_t^{c}) + \exp(rA_t^{f}) + \exp(rA_t^{0})}$$

$$w_t^{f} = \frac{\exp(rA_t^{f})}{\exp(rA_t^{c}) + \exp(rA_t^{f}) + \exp(rA_t^{0})}$$

and

$$\begin{aligned}\exp(p_{t+1}) - \exp(p_t) &= \exp\left(p_t + a\left(w_t^{c} b(p_t - x_t) + w_t^{f} c(f_t - p_t)\right)\right) \\ &\quad -\exp(p_t) \\ &= \exp(p_t)\left\{\exp\left(a\left(w_t^{c} b(p_t - x_t) + w_t^{f} c(f_t - p_t)\right)\right) - 1\right\}\end{aligned}$$

3.2 Steady State and Local Stability Analysis

At steady state, the dynamic variables turn out to be

$$p = f = x = y$$

and

$$A^{c} = A^{f} = A^{0} = 0$$

i.e. prices are at their fundamental levels and agents make no profits, so that the average realized profits (which measure the fitness of the rules) for each agent-type are zero in the long run. Therefore,

$$w^{c} = w^{f} = w^{0} = 1/3$$

implying that the agents are uniformly distributed among the three strategies. The local stability analysis of the steady state is performed via the localization, in the complex plane, of the eigenvalues of the Jacobian (evaluated at the steady state) of the map associated with the dynamical system. As it is known, a sufficient condition for the local asymptotic stability is that all the (real or complex) eigenvalues of the Jacobian lie inside the ‘unit circle’ in the complex plane, i.e. they are all smaller than one in modulus.

In the Appendix A it is shown that the Jacobian matrix evaluated at the steady state is block diagonal, which makes it possible to characterize analytically its eigenvalue structure, and that all of the eigenvalues are smaller than one in modulus if and only if the following set of inequalities is satisfied

This appendix contains the derivation of the Jacobian matrix of the map whose iteration determines the time evolution of the dynamical system (16), as well as the analysis of the eigenvalues of the Jacobian evaluated at the steady state. Denoting by ‘ ´ ’ the unit time advancement operator (that is; if *x* is the value of a variable at time *t*, then $\acute{x}$ is the value of the same variable at *t*+1), the dynamics of the system is obtained by iteration of the following 5-D map

$$G: \begin{cases} \acute{p} = p + a\left(w^c b(p-x) + w^f c(f-p)\right) \\ \acute{x} = p \\ \acute{y} = x \\ \acute{A^c} = ub(x-y) + mA^c \\ \acute{A^f} = uc(f-x) + mA^f \end{cases} \tag{16}$$

where

$$w^c = \frac{\exp(rA^c)}{z},$$

$$w^f = \frac{\exp(rA^f)}{z},$$

$$z = \exp(rA^c) + \exp(rA^f) + \exp(rA^0),$$

$$u = \exp(\acute{p}) - \exp(p)$$
$$= \exp(p)\left\{\left[a\left(w^c b(p-x) + w^f c(f-p)\right)\right] - 1\right\}$$

(i) The partial derivatives of $\acute{p}$ with respect to the variables p, x, and y gives;

$$\frac{\partial \acute{p}}{\partial p} = 1 + a(w^c b - w^f c)$$

and

$$\frac{\partial \acute{p}}{\partial x} = -a w^c b$$

which become at the steady state

$$\left.\frac{\partial \acute{p}}{\partial p}\right|_{s.s} = 1 + 0.33a(b - c)$$

and

$$\left.\frac{\partial \acute{p}}{\partial x}\right|_{s.s} = -0.33ab$$

and $\frac{\partial \acute{p}}{\partial y} = 0$.

(ii) Computing the partial derivatives of $\acute{p}$ with respect to the variables A^c and A^f reveals

$$\frac{\partial \acute{p}}{\partial v^c} = a\left(b(p - x)\frac{\partial w^c}{\partial A^c} + c(f - p)\frac{\partial w^f}{\partial A^c}\right)$$

and

$$\frac{\partial \acute{p}}{\partial A^f} = a\left(b(p - x)\frac{\partial w^c}{\partial A^f} + c(f - p)\frac{\partial w^f}{\partial A^f}\right)$$

Since at steady state $p = f = x$, these two partial derivatives will vanish at the steady state.

(iii) Calculating the partial derivatives of $\acute{A}^c$ and $\acute{A}^f$ with respect to the variables p, x, and y, which gives;

$$\frac{\partial \acute{A}^c}{\partial p} = b(x - y)\frac{\partial u}{\partial p},$$

$$\frac{\partial \acute{A}^f}{\partial p} = c(f - x)\frac{\partial u}{\partial p},$$

$$\frac{\partial \acute{A}^c}{\partial x} = b\left((x-y)\frac{\partial u}{\partial x} + u\right),$$

$$\frac{\partial \acute{A}^f}{\partial x} = c\left((f-x)\frac{\partial u}{\partial x} - u\right),$$

$$\frac{\partial \acute{A}^c}{\partial y} = -bu,$$

and

$$\frac{\partial \acute{A}^f}{\partial y} = 0$$

Also, all these partial derivatives vanish at the steady state as $p = f = x = y$ and $u = 0$.

(iv) The partial derivatives of $\acute{A}^c$ and $\acute{A}^f$ with respect to the variables A^c and A^f reveal the following.

$$\frac{\partial \acute{v}^c}{\partial A^c} = m,$$

$$\frac{\partial \acute{A}^f}{\partial A^f} = m$$

and

$$\frac{\partial \acute{v}^c}{\partial A^f} = \frac{\partial \acute{A}^f}{\partial A^c} = 0.$$

Taking the dynamic variables; p, x, y, A^c and A^f, one finds that the Jacobian matrix at the steady state (denoted by J) has the following block diagonal structure:

$$J = \begin{bmatrix} H & 0_{(3,2)} \\ 0_{(2,3)} & mI_2 \end{bmatrix}$$

where the matrix

$$H = \begin{bmatrix} 1 + 0.33a(b-c) & -0.33ab & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

And I_3 is the 3-D identity matrix, so that

$$mI_2 = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

Due to this particular structure, the eigenvalues can be obtained by computing separately the eigenvalues of each block H and mI_2. One gets immediately that two of the five eigenvalues are real and equal to *m* (and thus smaller than one in absolute value for$0 \leq m < 1$). Three of the eigenvalues are the ones of the block *H*. In turn, the 3-D matrix *H* is lower block triangular, with one of the eigenvalues equal to 0 (and thus smaller than one in modulus). The two further eigenvalues are the ones of the following 2-D block

$$Q = \begin{bmatrix} 1 + 0.33a(b-c) & -0.33ab \\ 1 & 0 \end{bmatrix}$$

Denote by $Tr(Q) = 1 + 0.33a(b-c)$ and $Det(Q) = 0.33ab$ the trace and the determinant of Q; respectively. The characteristic polynomial Q is given by $P(z) = z^2 - Tr(Q)z + Det(Q)$. A well-known necessary and sufficient condition (see e.g., Gandolfo, 1996) to have both eigenvalues smaller than one in modulus, which implies a locally attracting steady state, is the following:

$P(1) = 1 - Tr(Q) + Det(Q) > 0,$

$P(-1) = 1 + Tr(Q) + Det(Q) > 0,$

$P(0) = Det(Q) < 1.$

Conditions (17) could be rewritten in terms of the parameters as follows;

$0.33ac > 0,$

$$c < 6/a + 2b, \qquad (17)$$

$b < 3/a\,.$

Note that, stability features of the steady state do not depend on the other parameters.

4. Results and Analyses

This section displays the dynamics of our model by simulation. Subsection 4.1 describes the simulation design. The extent to which our model can explain statistical properties of real financial markets is investigated in Subsection 4.2. In addition to this, we run a Monte Carlo analysis to check the robustness of illustrated results.

4.1 Simulation Design

To implement the proposed parameter estimates for artificial financial market, we develop an agent-based simulation model using Netlogo platform [43]. At initialization, all parameters of the model are equal to the values defined in Table 1, and values of all variables; p_t, p_{t-1}, w_t^c ,w_t^f, w_t^0, D_{t-2}^c, D_{t-2}^f , D_t^c , D_t^f, A_{t-1}^c, A_{t-1}^f, A_t^c, and A_t^f are set to zero. We investigate the performance of 5000 simulation runs; each containing 4000 daily observations. In the following subsection, simulation results are displayed.

Table 1. Parameters for the simulation of the financial markets under loss aversion behavioural bias.

Parameter	Value	Description of parameter
a	1	Price settlement parameter
b	0.04	Extrapolating parameter
c	0.04	Reverting parameter
m	0.975	Memory parameter
r	300	Intensity of choice parameter
σ_α	0.01	Standard deviation of the random factors affecting the price settlement process
σ_β	0.05	Standard deviation of the additional random orders of technical trading
σ_γ	0.01	Standard deviation of the additional random orders of fundamental trading
λ	2.25	Loss aversion parameter

4.2 Simulation Results

In this subsection, important results are illustrated. Fig. 1. Depicts the price evolution, returns, and weights of one simulation run. Note that, prices are random walk, and they are fluctuating around the fundamental values. Returns possess the excess volatility and clustered volatility features. Finally, traders are fluctuating between the three trading strategies according to their payoffs.

To check the scaling behaviour of the artificial market under the estimated parameters, we compute the scaling exponent using detrended fluctuation analysis. Fig. 2 displays the estimation of the scaling exponent for raw H_r returns.

Note the linear relationship on a log-log scale between the average fluctuation $F(n)$, and the time scale, *n* indicates the presence of scaling in the time series at hand. The H_r yields a value of 0.43, which is close to that of the real financial markets. This result indicates white-noise processes.

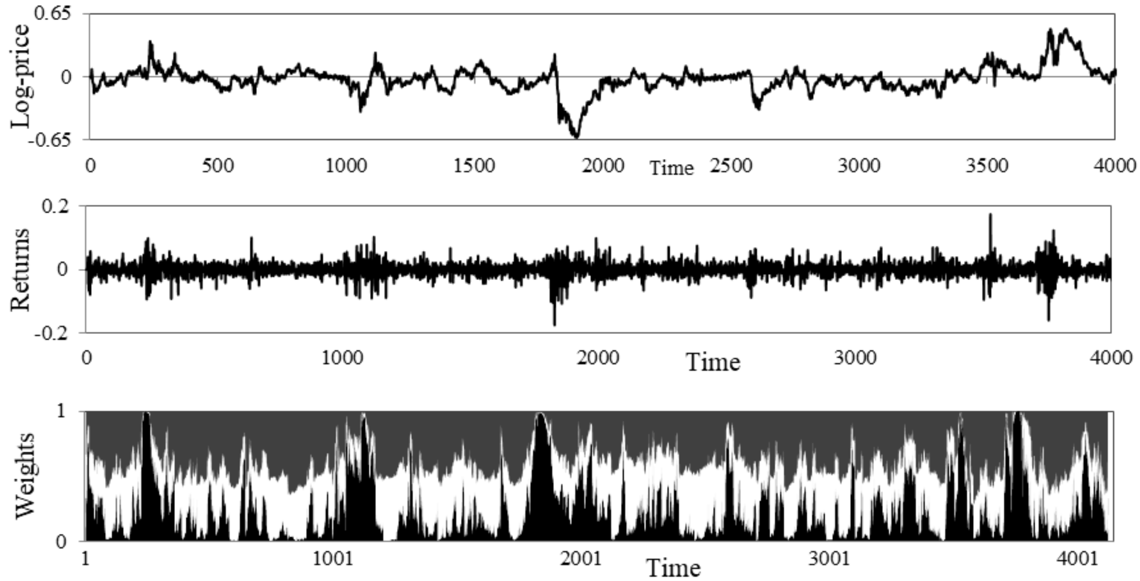

Fig. 1. Price evolution, returns, and weights of a simulation run.

Fig. 3 displays the estimation of the scaling exponent for absolute $H_{|r|}$ returns. Note linear relationship on a log-log scale between the average fluctuation $F(n)$, and the time scale, *n* indicates the presence of scaling in all the time series at hand.

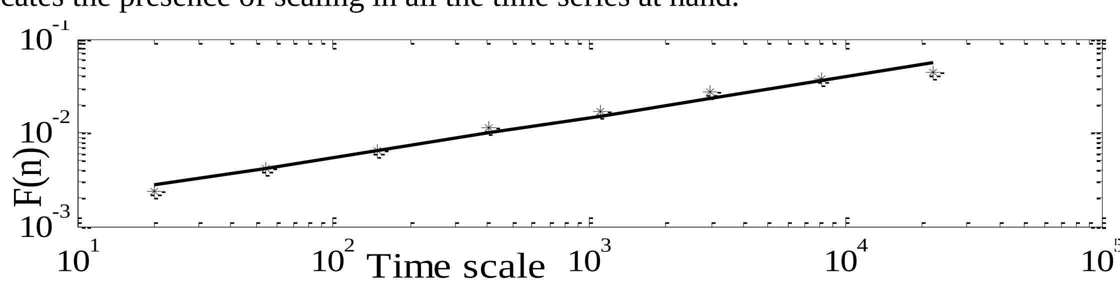

Fig. 2. Estimation of self-similarity parameter *H* for raw returns. To estimate the self-similarity parameter, we follow Peng et al. (1994) and perform a detrended fluctuation analysis (DFA). The slope of the line relating $\log(F(n))$ to $\log(n)$ is the estimated scaling exponent, $n = \{2^3, \ldots, 2^{10}\}$ The scaling exponent H_r yields a value of 0.43$\pm$0.054, which is close to the theoretically expected value of the white-noise process.

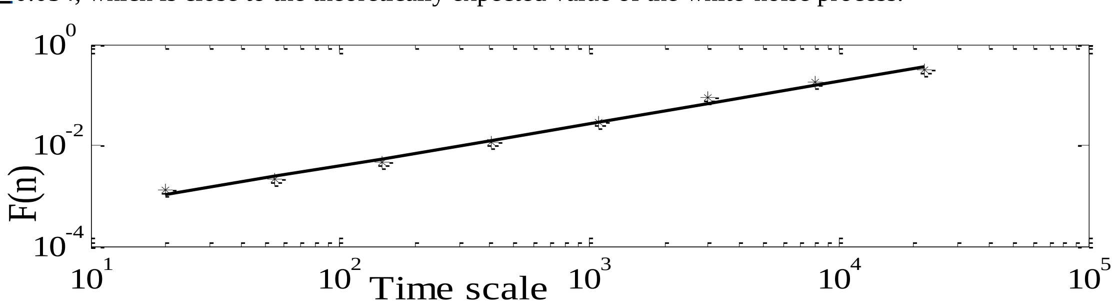

Fig. 3. Estimation of self-similarity parameter *H* for absolute returns. A linear relationship on a log-log scale plot indicates the presence of power-law scaling. The slope of the line relating $\log(F(n))$ to $\log(n)$ is the estimated scaling exponent, $n = \{2^3, \ldots, 2^{10}\}$ The scaling exponent $H_{|r|}$ reveals a value of 0.84$\pm$0.071, which indicates persistent long-range (power-law) autocorrelations in absolute returns for the time series under investigation.

The scaling exponent $H_{|r|}$ reveals a value of 0.84, which is in line with those for real financial markets, showing long-range power-law autocorrelations in absolute returns.

Now, we investigate the robustness of the scaling power-law. Table 2 reports the scaling exponent of raw returns H_r, and the scaling exponent of absolute returns $H_{|r|}$ for the estimates of the mean, the 5 percent, 25 percent, 50 percent, 75 percent, and 95 percent quantiles of these statistics.

For instance, H_r equals to 0.44 and 0.51for the lower and upper quantile, which are close to the results obtained for real market (Mandelbrot, 1963). This is in good agreement with absence of long memory in empirical financial returns and implies a small degree for predicting price changes. Moreover, estimates of $H_{|r|}$, for instance, hover between 0.80and 0.93 in 90 percent of the cases, which are close to the values reported for the financial (Mandelbrot, 1963). These values show persistent long-range autocorrelation in absolute return series.

Table 2. The scaling exponent for the raw and absolute returns, respectively. The table reports the scaling exponent of raw returns H_r, and the scaling exponent of absolute returns $H_{|r|}$ for the estimates of the mean, the 5 percent, 25 percent, 50 percent, 75 percent, and 95 percent quantiles of these statistics. Computations are based on 5000 time series, each containing 4000 observations.

Mean/ quantile	H_r	$H_{\|r\|}$
Mean	0.49	0.87
0.05	0.40	0.80
0.25	0.44	0.84
0.50	0.48	0.87
0.75	0.52	0.90
0.95	0.57	0.93

To sum up, the results show that the simulation with the proposed parameter estimates exhibit the stylized facts of real financial markets, such as random-walk prices, excess volatility, clustered volatility, and power-law tails.

5. Conclusion

Agent-Based Models (ABM) are able to capture important stylized facts of financial markets, such as clustered volatility, excess volatility, random-walk prices, fat-tailed returns distribution, power-law tails, and fractal structure. Accordingly, ABM were rapidly used in last decade as traditional economic models failed to explain complex behavior especially during and after crisis periods. Many models were introduced to simulate real behavior such as agents' irrational trading behavior. Selim et al. (Selim et al., 2015) proposed introducing loss-aversion behavior bias as an application of the prospect theory. The model performed very well in simulating the trading behavior in real financial markets. Thereby, estimating the model parameters is important to use it as a testbed by investors, researchers, and decision makers. In this paper, stability and bifurcation analysis is run to estimate the model's key parameters. Three conditions were reached to achieve the estimation process.

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