The Bayesian Estimation In step Partially Accelerated Life Tests For The Burr XII Parameters Using Type I Censoring

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Abstract

Some traditional life tests result in very few failures by the end of the test. So that to reach failure quickly test units are run at higher than usual stress conditions. This study is devoted to the estimating failure time data under step-stress partially accelerated life tests based on type I censoring. The lifetime distribution of the test items is assumed to follow Burr type XII distribution. The Bayesian estimates of the acceleration factor and the parameters of the lifetime distribution are obtained when the shape parameters and the acceleration factor are unknown. Numerical examples are given to study the posterior variances of the parameters and their mean square errors using Mathcad (2001). For illustrating the precision and variations of Bayesian estimators simulation results are included for different sample sizes.

Key words: Reliability; Step stress-Partially accelerated life test; Accelerated factor, Burr type XII distribution; Bayesian approach.

1. Introduction

Due to continuous improvement in manufacturing design, it is more difficult to obtain information about lifetime of products or materials with high reliability at the time of testing under normal conditions. This makes lifetime testing under these conditions is very costly and take a long time. To get the information about the lifetime distribution of these materials, a sample of these materials is subject to more severe operation conditions than normal ones. These conditions are called stresses which may be in the form of temperature, voltage, pressure, vibration, cycling rate, and load.

This kind of testing is called accelerated life test (ALT), where products are put under stresses higher than usual to yield more failure data in a short time. The life data from the high stresses are used to estimate the life distribution at design condition. There are mainly three ALT methods. The first method is called the constant stress ALT; the second one is referred to as step-stress ALT and the third is the progressive stress ALT. the first method is used when the stress remains unchanged, so that if the stress is weak, the test has to lose for a long time. The other two methods can reduce the testing time and save a lot of manpower, material sources and money (see Rao; (1992)). The major assumption in ALT is that the mathematical model relating the lifetime of the unit and the stress is known or can be assumed. In some cases, such life-stress relationships are not known and cannot be assumed, i.e ALT data can not be extrapolated to use condition. So, in such cases, partially accelerated life tests (PALT) is a more suitable test to be performed for which tested units are subjected to both normal and accelerated conditions.

According to Nelson (1990), the stress can be applied in various ways. One way is to accelerate failure in step-stress, which increases the stress applied to test product in a specified discrete sequence. Step-stress partially accelerated life test (SS-PALT) is used to get quick information for the lifetime of product with high reliability; especially, when the mathematical model related to test conditions of mean lifetime of the product is unknown and can not be assumed. The step stress scheme applies stress to test units in the way that the stress will be changed at prespecified time. Generally, a test unit starts at a specified low stress. If the unit does not fail at a specified time, stress on it raised and held at a specified time. . Stress is repeatedly increased until test unit fails or censoring time is reached.

For an overview of SS-PALT, there is an amount of literature on designing SS- PALT. Goel (1971) considered the estimation problem of the accelerated factor using both maximum likelihood and Bayesian methods for items having exponential and uniform distributions. DeGroot and Goel (1979) estimated the parameters of the exponential distribution and acceleration factor in SS-PALT using Bayesian approach, with different loss functions. Also, Bhattacharyya and Soejoeti (1989) estimated the parameters of the Weibull distribution and an acceleration factor using maximum likelihood method. Bai and Chung (1992) estimated the scale parameter and acceleration factor for exponential distribution under type I censored sample using maximum likelihood method.

Attia *et al* (1996) considered the maximum likelihood method for estimating the acceleration factor and the parameters of Weibull distribution in SS-PALT under type I censoring. Abel-Ghaly *et al* (1997) used Bayesian approach for estimating the parameters of Weibull distribution parameters with known shape parameter. They studied the estimation problem in SS-PALT under both type I and type II censored data. Abdel-Ghani (1998) considered the estimation problem of the parameters of Weibull distribution and the acceleration factor for both SS-PALT and constant-stress PALT. Maximum likelihood and Bayesian methods under type I and type II censored data are applied in this study. Abdel-Ghaly *et al* (2002a) studied the estimation problem of the acceleration factor and the parameters of Weibull distribution in SS-PALT using maximum likelihood method in two types of data, namely type I and type II censoring. Abdel-Ghaly *et al* (2002b, 2003) studied both the estimation and optimal design problems for the Pareto distribution under SS-PALT with type I and type II censoring. Abdel-Ghani (2004) considered the estimation problem of log-logistic distribution parameters under SS-PALT.

Recently, Ismail (2004) used maximum likelihood and Bayesian methods for estimating the acceleration factor and the parameters of Pareto distribution of the second kind. Ismail (2006) studied the estimation and optimal design problems for the Gompertz distribution in SS-PALT with type I censored data. Abd-Elfattah *et al* (2007, 2008) studied the estimation problem of the acceleration factor and the parameters of Brr type XII distribution in SS-PALT using maximum likelihood method under type I and type II censoring.

This article is devoted to the Bayesian approach when applied for the estimation problem in the case of step-stress PALT under squared error loss function using Jaffreys vague prior or the non-informative prior (NIP) with type I censored sampling. Also, the posterior variances of the parameters are given. Numerical examples are given to study the posterior variances of the parameters and their mean square errors.

This article is organized as follows. In section 2 the Burr type XII distribution is introduced as a lifetime model and the test method is also described. Section 3 presents estimators of parameters and acceleration factor for the Burr type XII under type I censoring using Bayesian approach. Section 4 is the simulation studies for illustrating the theoretical results. Finally, short conclusions are included.

2. The Model and Test Model

This section introduces the assumed model for product life and also fully describes the test method.

Notation:

ALT	accelerated life testing.
OALT	ordinary accelerated life testing.
PALT	partially accelerated life testing.
SS-PALT	step-stress partially accelerated life testing.
n	total number of test items in a PALT.
T	lifetime of an item at normal conditions.
Y	total lifetime of an item in a SS-PALT.
y_i	observed value of the total lifetime Y_i of item i , $i=1,...,n$.
η	censoring time of a PALT.
$f(t)$	probability density function at time t at normal conditions.
$F(t)$	cumulative distribution function at time t at normal conditions.
$R(t)$	reliability function at time t at normal conditions.
β	acceleration factor $(\beta > 1)$.
τ	stress change time in a SS-PALT ($\tau < y_{(r)}$).
c, k	the shape parameters of the Burr type XII distribution.
δ_{1i}, δ_{2i}	indicator functions in a SS-PALT: $\delta_{1i} = I(Y_i \le \tau)$, $\delta_{2i} = I(\tau < Y_i \le y_{(r)})$.
n_u, n_a	number of items failed at normal and accelerated conditions respectively.
$L(.)$	likelihood function.
NIP	non-informative prior.
$u(.)$	prior distribution.
$g(.)$	posterior distribution.
$y_{(1)} \le ... \le y_{(n_u)} \le \tau \le y_{(n_u+1)} \le ... \le y_{(r)}$	ordered failure times at the two conditions.
MSE	mean square error.
RABias	absolute relative bias.
RE	relative error.

2.1 The Burr type XII distribution: As a Failure Time Model

As a member of the Burr (1942) family of distributions, this includes twelve types of cumulative distribution functions with a variety of density shapes. The two parameter Burr type XII distribution denoted by Burr (c,k) has density function of the form

$$f(t,c,k) = c\,k\,t^{c-1}\left(1+t^{c}\right)^{-(k+1)}, \qquad t > 0,\ c > 0 \text{ and } k > 0 \tag{2.1}$$

where, c and k are the shape parameters of the distribution.

The cumulative distribution function is

$$F(t,c,k) = 1-\left(1+t^{c}\right)^{-(k)}, \qquad t > 0,\ c > 0 \text{ and } k > 0 \tag{2.2}$$

The reliability function of the Burr type XII distribution is given by:

$$R(t,c,k) = \left(1+t^{c}\right)^{-(k)} \tag{2.3}$$

The Burr(c,k) distribution was first proposed as a lifetime model by Dubey (1972, 1973). Evans and Simons (1975) studied further the distribution as a failure model and they also derived the maximum likelihood estimators as well moments of the Burr(c,k) probability density function. Lewis (1981) noted that the Weibull and exponential distributions are special limiting cases of the parameter values of the Burr(c,k) distribution. She proposed the use of the Burr(c,k) distribution as a model in accelerated life test data.

2.2 The test method

In SS-PALT, all of the n units are tested first under normal condition, if the unit does not fail for a prespecified time τ, then it runs at accelerated condition until failure. This means that if the item has not failed by some prespecified time τ, the test is switched to the higher level of stress and it is continued until items fails. The effect of this switch is to multiply the remaining lifetime of the item by the inverse of the acceleration factor β. In this case the switching to the higher stress level will shorten the life of test item. Thus the total lifetime of a test item, denoted by Y, passes through two stages, which are the normal and accelerated conditions. Then the lifetime of the unit in SS-PALT is given as follows:

$$Y = \begin{cases} T & if\, T \le \tau \\ \tau + \beta^{-1}(T-\tau) & if\, T > \tau, \end{cases} \quad (2.4)$$

where, T is the lifetime of an item at use condition, τ is the stress change time and β is the acceleration factor which is the ratio of mean life at use condition to that at accelerated condition, usually $\beta > 1$. Assume that the lifetime of the test item follows Burr type XII distribution with shape parameters c and k. Therefore, the probability density function of total lifetime Y of an item is given by:

$$f(y) = \begin{cases} 0 & if\;\; y \le 0 \\ f_1(y) & if\;\; 0 < y \le \tau \\ f_2(y) & if\;\; y > \tau \end{cases} \quad (2.5)$$

where, $f_1(y) = c\,k\,y^{c-1}\left(1+y^c\right)^{-(k+1)}\; c,k>0$, is the equivalent form to equation (2.1), and,

$f_2(y) = \beta\, c\, k\left[\tau + \beta(y-\tau)\right]^{c-1}\left[1 + \{\tau + \beta(y-\tau)\}^c\right]^{-(k+1)}, c,k>0,\; \beta>1$, is obtained by the transformation variable technique using equations (2.1) and (2.4).

3. The Bayesian estimates under type I censoring

In Bayesian approach, the parameter of the life time distribution are treated as random variables. The prior information about these unknown parameters which is due to the properties of materials and the knowledge of engineering facts may be presented in measureable form as a prior distribution. Then, Bayesian analysis allows incorporating expert knowledge in measureable form as a prior distribution for the parameters with the data observed from the experiments. The posterior information are obtained to anticipate the behaviour of products in the future. Consequently, the importance of applying the Bayesian analysis in partially accelerated life tests is of great practical interest. in this section, Bayesian estimates and posterior variances of the acceleration factor and the shapes parameters are obtained in the case of type I censoring. Moreover, for illustration, numerical examples are given.

Considering the case of type I censoring, then the likelihood function of the total lifetimes $y_1, \ldots, y_n$, of n items independent and identically distributed random variables takes the following form

$$L(\underline{y}|\,\beta,c,k)=\prod_{i=1}^{n}\{f_1(y_i)\}^{\delta_{1i}}\{f_2(y_i)\}^{\delta_{2i}}\{R(\eta)\}^{\bar{\delta}_{1i}\bar{\delta}_{2i}}$$

$$\begin{aligned}L(\underline{y}|\,\beta,c,k)=\prod_{i=1}^{n}&\left\{c\,k\,y_i^{\,c-1}\left(1+y_i^{\,c}\right)^{-(k+1)}\right\}^{\delta_{1i}}\\&\times\left\{c\,k\,\beta\left[\tau+\beta\left(y_i-\tau\right)\right]^{c-1}\left[1+\left(\tau+\beta\{y_i-\tau\}\right)^c\right]^{-(k+1)}\right\}^{\delta_{2i}}\\&\times\left\{1+\left[\tau+\beta(\eta-\tau)\right]^c\right\}^{-k\bar{\delta}_{1i}\bar{\delta}_{2i}}\end{aligned}$$

$$L(\underline{y}|\,\beta,c,k)=(c\cdot k)^{n_0}(\beta)^{n_a}Q(\beta,c,k) \tag{3.1}$$

where,

$$Q(\beta,c,k)=\left\{\prod_{i=1}^{n_u}(y_i)^{c-1}\left(1+y_i^c\right)^{-(k+1)}\right\}\times\left\{\prod_{i=n_{u+1}}^{n_0}(A)^{c-1}\left[1+(A)^c\right]^{-(k+1)}\right\}\times\left\{1+(D)^c\right\}^{-k[n-n_0]},$$

$A=[\tau+\beta\{y_i-\tau\}]$, $D=[\tau+\beta(\eta-\tau)]$, $\sum_{i=1}^{n}\delta_{1i}=n_u$, $\sum_{i=1}^{n}\delta_{2i}=n_a$,

$\sum_{i=1}^{n}\bar{\delta}_{1i}\bar{\delta}_{2i}=n-n_u-n_a$ and $n_0=n_u+n_a$

It is assumed that the parameters are a prior independent, and let the non-informative prior (NIP) for each parameter be represented. It follows that a NIP for c,k are respectively given by

$$u_1(c)\propto c^{-1},\qquad c>0$$

$$u_2(k)\propto k^{-1},\qquad k>0$$

and the NIP's for β is given by

$$u_3(\beta)\propto\beta^{-1},\qquad \beta>1$$

consequently, the joint NIP is given by taking the product of three NIP's of parameters as follows

$$u(\beta,c,k)=(\beta\,c\,k)^{-1},\qquad c,k>0\quad and\quad \beta>1. \tag{3.2}$$

Multiplaying (3.1) by (3.2), the joint posterior density of β,c and k given the data is shown to be

$$g(\beta,c,k\,|\,\underline{y})=Q_2/Q_1,\qquad c,k>0\quad and\quad \beta>1 \tag{3.3}$$

where, Q_1 is normalized constant equal to $Q_1=\int_0^\infty\int_0^\infty\int_1^\infty L(\underline{y}|\,\beta,c,k)\cdot u(\beta,c,k)\;d\beta\,dk\,dc$

and $Q_2=(c\cdot k)^{n_0-1}(\beta)^{n_a-1}Q(\beta,c,k)$.

Now, marginal posterior of any parameter is obtained by integrating the joint posterior distribution with respect to other parameters. The posterior probability density function of c can be written as

$$g_1(c|\beta,k,\underline{y}) = Q_4/Q_1, \qquad c>0 \tag{3.4}$$

where, $Q_4 = (c)^{n_0}\left\{\prod_{i=1}^{n_u}(y_i)^{c-1}\right\}\cdot\int_0^\infty\int_1^\infty(k)^{n_0-1}(\beta)^{n_a-1}\cdot Q_3 \qquad d\beta\, dk$

and $Q_3 = \left\{\prod_{i=1}^{n_u}\left(1+y_i^c\right)^{-(k+1)}\right\}\times\left\{\prod_{i=n_u+1}^{n_0}(A)^{c-1}\left(1+[A]^c\right)^{-(k+1)}\right\}\times\left\{1+(D)^c\right\}^{-k[n-n_0]}.$

Similarly integrating the joint posterior with respect to c and β, the marginal posterior of k can be obtained as

$$g_2(k|\beta,c,\underline{y}) = Q_5/Q_1, \qquad k>0 \tag{3.5}$$

where, $Q_5 = k^{n_0-1}\int_0^\infty\int_1^\infty c^{n_0-1}\beta^{n_a-1}\cdot Q \qquad d\beta\, dc,$

and finally, the marginal posterior of β can be obtained as

$$g_3(\beta|c,k,\underline{y}) = Q_6/Q_1, \qquad \beta>1 \tag{3.6}$$

where, $Q_6 = \beta^{n_a-1}\int_0^\infty\int_0^\infty c^{n_0-1}k^{n_0-1}\cdot Q \qquad dc\, dk$.

It is well known that under a squared error loss function, the Bayes estimator of a parameter is its posterior mean. To obtain the posterior mean and posterior variance of the acceleration factor, and shapes parameters, non-tractable integrals will be confronted. So, in this problem numerical integration is required. Then, both the posterior mean and posterior variance of the shapes parameters (c,k) and the acceleration factor are expressed as follows

$$E(c|\beta,k,\underline{y}) = \tilde{c} = Q_7/Q_1 \tag{3.7}$$

$$\mathrm{var}(\tilde{c}|\beta,k,\underline{y}) = Q_8/Q_1 \tag{3.8}$$

$$E(k|\beta,c,\underline{y}) = \tilde{k} = Q_9/Q_1 \tag{3.9}$$

$$\mathrm{var}(\tilde{k}|\beta,c,\underline{y}) = Q_{10}/Q_1 \tag{3.10}$$

$$E(\beta|c,k,\underline{y}) = \tilde{\beta} = Q_{11}/Q_1 \tag{3.11}$$

$$\text{var}(\tilde{\beta}|c,k,\underline{y}) = Q_{12}/Q_1 \tag{3.12}$$

where,

$$Q_7 = \int_0^\infty\int_0^\infty\int_1^\infty c \cdot Q_2 \quad d\beta\, dc\, dk, \quad Q_8 = \int_0^\infty\int_0^\infty\int_1^\infty (\tilde{c}-c)^2 \cdot Q_2 \quad d\beta\, dc\, dk,$$

$$Q_9 = \int_0^\infty\int_0^\infty\int_1^\infty k \cdot Q_2 \quad d\beta\, dc\, dk, \quad Q_{10} = \int_0^\infty\int_0^\infty\int_1^\infty (\tilde{k}-k)^2 \cdot Q_2 \quad d\beta\, dc\, dk,$$

$$Q_{11} = \int_0^\infty\int_0^\infty\int_1^\infty \beta \cdot Q_2 \quad d\beta\, dc\, dk \quad \text{and} \quad Q_{12} = \int_0^\infty\int_0^\infty\int_1^\infty (\tilde{\beta}-\beta)^2 \cdot Q_2 \quad d\beta\, dc\, dk$$

Since, these equations (5.7) to (5.12) are very difficult to obtain. An iterative procedure is applied to solve these equations numerically using Mathcad (2001) statistical package to obtain the posterior mean and posterior variance of the shapes parameters (c,k) and the acceleration factor.

4. Simulation Studies

Simulation studies have been performed using Mathcad (2001) for illustrating the theoretical results of the estimation problem. The performance of the resulting estimators of the acceleration factor and two shape parameters has been considered in terms of their absolute relative bias (RABias), mean square error (MSE), and relative error (RE). The Simulation procedures were described below:

Step 1: 1000 random samples of sizes 10, 20 and 30 were generated from Burr type XII distribution. The generation of Burr type XII distribution is very simple, if U has a uniform (0,1) random number, then $Y = [(1-U)^{(\frac{-1}{k})} - 1]^{(\frac{1}{c})}$ follows a Burr type XII distribution. The true parameters selected values are $(c = 1, \beta = 1.25, k = 2)$.

Step 2: Choosing the censoring time τ at the normal condition to be $\tau = 3$ and censoring time of a PALT to be $\eta = 6$.

Step 3: For each sample and for the two sets of parameters, the acceleration factor and the parameters of the distribution were estimated in SS-PALT under type II censored sample.

Step 4: Newton Raphson method was used for solving the equations (5.7) to (5.12) to obtain the posterior mean and posterior variance of the shape parameters (c,k) and the acceleration factor.

Step 5: The RABias, MSE, and RE of the estimators for the acceleration factor and the two shape parameters for all sample sizes for the parameters were tabulated.

Simulation results are summarized in Tables 7. Table 7 gives the posterior mean and posterior variance of the shape parameters (c,k) and the acceleration factor and the RABias, MSE, and the RE of the estimators.

Appendix

Table : The posterior mean, the posterior variance, RABias, MSE and RE for different sized samples of $c=1$, $\beta=1.25$ and $k=2$ given $\tau=3$ and $\eta=6$ using type I censoring

n	Parameters (c,β,k,τ)	Posterior Mean	Posterior Variance	RABias	MSE	RE
10	c	0.46	74.745	0.54	75.037	0.54
	β	2.818	409.15	1.254	411.61	1.568
	k	1.794	431.15	0.275	431.19	0.206
20	c	0.015	7.663	0.985	8.633	0.985
	β	0.094	15.464	0.925	16.80	1.156
	k	0.060	49.221	2.587	52.99	1.94
30	c	0.0003	0.157	1.00	1.157	1.00
	β	0.0019	0.346	0.99	1.904	1.248
	k	0.0012	1.014	2.67	5.009	1.999

We note that, increasing sample size decreasing MSE for all parameters.

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