A Modified Anderson Darling Goodness of Fit Test for the Generalized Rayleigh Distribution

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Abstract

A modified Anderson Darling goodness of fit test is constructed for the generalized Rayleigh distribution in the case where the parameters are estimated from the sample. Critical values are generated using a Monte Carlo simulation procedure with 10,000 repetitions each. Random samples of 10 different sizes were drawn from generalized Rayleigh distribution with different values of shape parameter. Fitted response functions for the critical values based on the sample size and the shape parameter values are reported to avoid using a vast array of tables. This is followed by a power comparison using Monte Carlo methods at 5% and 1% significance levels for different sample sizes.

Keywords

Anderson Darling test; Generalized Rayleigh; Maximum likelihood; Monte Carlo simulation; Power function

1. Introduction

The Rayleigh distribution is widely used to model events that occur in different fields such as medicine, social and natural sciences. For instance, it is used in the study of various types of radiations, such as sound and light measurements. It is also used as a model for wind speed and is often applied to wind driven electrical generation.

In recent years, several standard life time distributions have been generalized via exponentiation. Examples of such exponentiated distributions are the exponentiated Weibull family, the exponentiated exponential, the exponentiated (generalized) Rayleigh, and the exponentiated Pareto family of distributions.

Amongst the authors who have considered the exponentiated distributions are Mudholkar and Hutson (1996), Gupta and Kundu (2001), Surles and Padgett (2001) and Kundu and Gupta (2007, 2008). A common feature in families of exponentiated distributions is that the distribution function may be written as $F(x)=[G(x)]^{\alpha}$, where $G(.)$, is the distribution function of a corresponding non generalized distribution and $\alpha>0$ denotes the generalized parameter. The generalized Rayleigh distribution is obtained by generalization of the Rayleigh distribution. It is also called the two parameter (scale and shape) Burr type X distribution.

The generalized Rayleigh density function is always right skewed, so it can be used quite effectively to analyze skewed data set. The one parameter (scale parameter equals one) generalized Rayleigh distribution is studied by Sartawi and Abu-Salih(1991), Jaheen (1995, 1996), Ahmad et al. (1997), Raqab (1998) and Surles and Padgett (1998). Recently Surles and Pudgett (2001) observed that the two parameter generalized Rayleigh distribution can be used quite effectively in modeling strength and general life time. Kundu and Raqab (2005) used different methods to estimate the unknown parameters of the generalized Rayleigh. Raqab and Kundu (2006) discuss several interesting properties of the generalized Rayleigh distribution. Figure (1) shows the probability density functions of the generalized Rayleigh distribution for shape parameter α =0.25, 0.5, 1, 2 *and* 10.

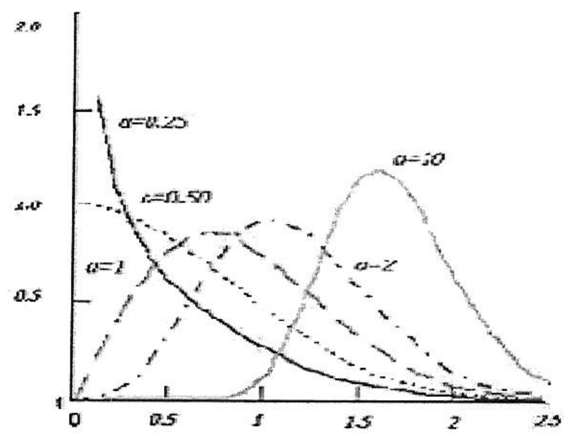


Figure (1): The density functions of the generalized Rayleigh distribution for different values of shape parameter

Goodness of fit techniques means the methods of examining how well a sample of data agrees with a given distribution as its population. The important goodness of fit tests are: (i) Tests of chi square types. (ii) Moment ratio techniques. (iii) Tests based on correlation. (iv) Tests based on empirical distribution function (EDF).

Most of these test statistics suffer from serious limitations. The most popular being the Pearson chi square test. However, the chi square test involves grouping of data which in the case of small samples results in a loss of the valuable information. The distribution theory of chi square statistics is a large sample theory. It is easy to use, but it is generally less powerful than EDF tests (D'Agostino and Stephens (1986)). The higher order moments are usually under estimated and this fact prevents the use of moment ratio techniques and so would be the case with correlation type tests.

Several power studies have mentioned that EDF tests are more powerful than other tests of fit for a wide range of sample sizes. Recently, the use of EDF tests has been difficult due to lack of available tables of significance points for the case of unknown parameters of the assumed distribution. This case is referred by Stephens (1974, 1977). The significance points that have been available are appropriate to the case where the parameters of the distribution are known. Such tables are of limited value in practice because the parameters of the distribution are seldom known. When the parameters are estimated, the critical values are considerably smaller than for the specified parameter case. The most important EDF tests are Kolmogorov Smirnov (KS), Cramer von Mises (CVM) and Anderson Darling (AD) tests. Stephens (1976) has shown in a wide variety of situations that AD is the most powerful EDF test. For assessing the behavior of the lower and upper tail of generalized Rayleigh distribution, a modified AD test is included in this study.

Using simulation methods this article generates asymptotic percentage points of the modified AD goodness of fit test statistic for the generalized Rayleigh distribution. The focus is on the most practical case when the parameters are unknown and have to be estimated from the sample. The article is arranged as follows. In section 2, the parameter estimation method is presented. In section 3, the modified AD test is discussed. While section 4, contains the response functions that give the percentage points for AD test statistic and the sampling distribution for AD statistic is obtained. Section 5, contains an example employing data. Finally, in section 6, we will provide a power comparison study using Monte Carlo simulation.

2. Estimating the parameters of the generalized Rayleigh Distribution

Let X be a random variable with scale parameter $\beta > 0$ and shape parameter $\alpha > 0$, the cumulative distribution function (CDF) of X is given by

$$F(x,\alpha,\beta) = (1-e^{-(\beta x)^2})^{\alpha}, \quad x > 0 \tag{1}$$

Therefore the probability density function (pdf) has the form

$$f(x,\alpha,\beta) = 2\alpha\beta^2 x e^{-(\beta x)^2}(1-e^{-(\beta x)^2})^{\alpha-1}, \quad x > 0 \tag{2}$$

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from generalized Rayleigh, then the log likelihood function can be written as;

$$L = c + n\ln(\alpha) + 2n\ln(\beta) + \sum_{i=1}^{n}\ln(x_i) - \beta^2\sum_{i=1}^{n}x_i^2 + (\alpha-1)\sum_{i=1}^{n}\ln(1-e^{-(\beta x_i)^2}) \tag{3}$$

where c is constant.

The normal equations become,

$$\frac{\partial L}{\partial \alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\ln(1-e^{-(\beta x_i)^2})=0 \tag{4}$$

$$\frac{\partial L}{\partial \beta}=\frac{2n}{\beta}-2\beta\sum_{i=1}^{n}x_i^2+2\beta(\alpha-1)\sum_{i=1}^{n}\frac{x_i^2 e^{-(\beta x_i)^2}}{1-e^{-(\beta x_i)^2}}=0 \tag{5}$$

Then the maximum likelihood estimation of α as a function of β can be obtained from (4) as

$$\hat{\alpha}(\beta)=\frac{-n}{\sum_{i=1}^{n}\ln(1-e^{-(\beta x_i)^2})} \tag{6}$$

Substituting (6) in (5), and solve for β to get the maximum likelihood estimation $\hat{\beta}$ of β as a solution of $h(\lambda)=\lambda$ where

$$h(\lambda)=\left[\frac{\sum_{i=1}^{n}\frac{x_i^2 e^{-\lambda x_i^2}}{(1-e^{-\lambda x_i^2})}}{\sum_{i=1}^{n}\ln(1-e^{-\lambda x_i^2})}+\frac{1}{n}\sum_{i=1}^{n}\frac{x_i^2}{(1-e^{-\lambda x_i^2})}\right]^{-1} \tag{7}$$

If $\hat{\lambda}$ is a solution of (7), hence $\hat{\beta}=\sqrt{\hat{\lambda}}$, see Kundu and Raqab (2005).

The maximum likelihood estimation $\hat{\alpha}$ of α is exact by (6) after replacing β by $\hat{\beta}$. Note that $\hat{\alpha}$ *and* $\hat{\beta}$ are not in explicit form. So iterative procedure will be used to obtain $\hat{\beta}$ using (7) and directly obtain $\hat{\alpha}$ using (6). The IMSL subroutines for generating random numbers and Newton Raphson technique for solving the nonlinear equation (7) have been used.

3. The modified AD test statistic

The goodness of fit problem can be stated as follows: test the null hypothesis, H_0, that the random sample $X_1, X_2, \ldots, X_n$ of size n, with order statistics $X_{(1)} \le X_{(2)} \le \ldots \le X_{(n)}$ is generated by a particular distribution. Anderson and Darling (1954) proposed an EDF test statistic as: $A_n^2 = n \int_{-\infty}^{\infty} \{F_n(x) - F(x)\}^2 \psi(x)\, dF(x)$

Where $\psi(x) = F(x)[1 - F(x)]^{-1}$ is a weight function, and $F_n(x)$ is the EDF of the random sample $X_1, X_2, \ldots, X_n$ with size n. Sinclair et al. (1990) proposed a modified form of the AD test statistic using the weight function $\psi(x) = [1 - F(x)]^{-1}$ as

$$V_n^2 = n \int_{-\infty}^{\infty} \{F_n(x) - F(x)\}^2 \psi^2(x)\, dF(x)$$

Choulakian and Stephens (2001), D'Agostino and Stephens (1986) and Stephens (1976) demonstrated that it is the most powerful among a wide set of available tests in the case of small samples.

For computational purpose, AD and modified AD statistics can be written in the series forms:

$$A_n^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\{\ln(z_i + \ln(1 - z_{n+1-i})\} \qquad (8)$$

$$V_n^2 = \frac{n}{2} - 2\sum_{i=1}^{n} z_i - \sum_{i=1}^{n}\left\{2 - \frac{(2i-1)}{n}\right\}\{\ln(1 - z_i)\} \qquad (9)$$

Where $z_i = F(x_{(i)}),\ i = 1, 2, \ldots, n$.

The detailed steps for testing H_0 involve: (i) the arrangement of data in ascending order; (ii) the calculation of the CDF given in (1) by using the estimated parameters; and (iii) the calculations of the A_n^2 *and* V_n^2 statistics values using equations (8) and (9).

(iv) Finally, if the values of the statistics exceed the critical values of A_n^2 *or* V_n^2 at a particular level of significance the null hypothesis is rejected.

4. The percentage points of A_n^2 and V_n^2

In order to determine the percentage points of A_n^2 *and* V_n^2 statistics, Monte Carlo simulations were conducted for sample sizes of $n = 10(5)40, 50, 70, 100$ and for each value of the shape parameter $\alpha = 0.25, 0.5, 1, 2, 10$. Without loss of generality the scale parameter β is set equal to 1, since the estimator of the scale parameter is scale invariant (see, Kundu and Raqab (2005)). For each combination of sample size and shape value, 10,000 samples from the generalized Rayleigh were simulated. The parameters were estimated by using maximum likelihood method and the A_n^2 *and* V_n^2 values were calculated by using equations (8) and (9) respectively. Finally, the A_n^2 *and* V_n^2 values were ranked and the $50^{th}, 75^{th}, 85^{th}, 90^{th}, 95^{th}$, *and* 99^{th} percentiles were found. In order to avoid using a large number of tables of critical values, response functions were estimated which give the A_n^2 *or* V_n^2 for each combination of sample size (n) and shape parameter values (α) at the $50\%, 25\%, 15\%, 10\%, 5\%$, *and* 1%.

Tables (1) and (2) are obtained using the software package MathCAD (2001) and the critical values of the two test statistics A_n^2 & V_n^2. The procedure was as follows:

Obtain a random sample of size n from generalized Rayleigh varieties at the specified value of shape parameter. This random sample is used to calculate the test statistics A_n^2 & V_n^2. Repeat this process 10,000 times. Hence, we aim to find the best fit function between critical values as dependent variable & the sample size and value of shape parameter as independent variables. So that the equations (10) and (11) are obtained with estimated coefficients $\hat{b}_o, \hat{b}_1, \hat{b}_2, \hat{b}_3, \hat{b}_4, \hat{b}_5$ for equation (10) and the estimated coefficients $\hat{c}_o, \hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4, \hat{c}_5$ for equation (11).

The response functions for the critical values A_n^2 *and* V_n^2 are obtained in the following equations:

$$y_\gamma = b_0 + b_1(n)^{-1} - b_2(n)^{-2} - b_3(\alpha) + b_4(\alpha)^{-2} + b_5(\alpha)^{-3} \quad (10)$$

$$u_\gamma = c_0 - c_1(n)^{-1} + c_2(n)^{-2} + c_3(n)^{-3} + c_4(\alpha) + c_5(\alpha)^{-2} \quad (11)$$

where γ is the level of significance, y_γ and u_γ are the critical values of the A_n^2 *and* V_n^2 test statistics respectively. The sample size is denoted by n and α is the shape parameter.

We can use equation (10) to obtain the critical value, y, for A_n^2 at specified n, α and significance level γ. For example, when γ = 50% , n =10 and α = 2 the coefficients $\hat{b}_o = .241, \hat{b}_1 = .213, \hat{b}_2 = .431, \hat{b}_3 = .105, \hat{b}_4 = .243, \hat{b}_5 = .462$ will be used in equation (10) to find the critical value of A_n^2 .

The estimated coefficients of the response functions of A_n^2 *and* V_n^2, their t-statistic values and values of R^2 are given in Tables (1) and (2) respectively.

The critical values A_n^2 *and* V_n^2 calculated from the sample at particular significance level using the sample size and the shape parameter value. If the calculated values of A_n^2 *or* V_n^2 exceed the corresponding critical value, the null hypothesis is rejected at the particular significance level.

Table (1)

Response functions for the A_n^2 critical values for both shape parameter values and all sample sizes at 50%, 25%, 15%, 10%, 5%, *and* 1% significance levels

Signf. Level (γ)	Coefficient estimates						
	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$	$\hat{b}_4$	$\hat{b}_5$	R^2
50%	.241 (53.7)	.213 (8.33)	.431 (6.77)	.105 (17.3)	.243 (22.8)	.462 (42.3)	.983
25%	.336 (44.2)	.264 (5.53)	.627 (6.21)	.123 (15.8)	.257 (23.4)	.537 (40.2)	.981
15%	.452 (31.6)	.352 (4.77)	1.341 (5.44)	.216 (14.6)	.317 (25.7)	.742 (37.8)	.972
10%	.483 (26.8)	.397 (4.21)	2.517 (4.82)	.243 (12.9)	.416 (27.2)	.919 (36.6)	.972
5%	.522 (19.3)	.426 (3.02)	3.102 (3.11)	.271 (11.2)	.527 (27.9)	1.324 (34.4)	.966
1%	.601 (11.5)	.864 (2.89)	4.826 (2.17)	.284 (9.37)	.546 (29.5)	1.639 (32.7)	.961

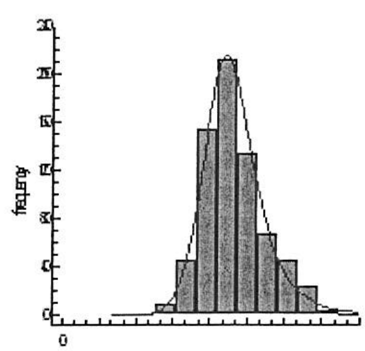
Table (2)

Response functions for the V_n^2 critical values for both shape parameter values and all sample sizes at 50%, 25%, 15%, 10%, 5%, *and* 1% significance levels

Signf. Level (γ)	Coefficient estimates						
	$\hat{c}_0$	$\hat{c}_1$	$\hat{c}_2$	$\hat{c}_3$	$\hat{c}_4$	$\hat{c}_5$	R^2
50%	.532 (15.7)	.662 (17.3)	1.105 (22.7)	.752 (9.23)	.333 (144.1)	.622 (36.7)	.991
25%	.591 (12.2)	.681 (14.5)	1.273 (20.1)	.523 (9.11)	.571 (132.4)	.701 (33.1)	.985
15%	.613 (11.6)	.742 (13.7)	1.471 (18.2)	.362 (8.64)	.722 (125.5)	.822 (31.6)	.981
10%	.720 (9.80)	.775 (11.1)	1.817 (17.1)	.237 (7.23)	.966 (122.2)	.909 (28.8)	.977
5%	.762 (9.30)	.810 (9.27)	2.551 (15.5)	.211 (6.55)	1.217 (119.1)	1.224 (26.1)	.970
1%	.789 (7.15)	.844 (8.90)	3.621 (12.7)	.178 (5.13)	1.542 (117.3)	1.391 (25.5)	.965

Table (3)
The Sampling Distribution of A_n^2 and V_n^2

n	Test Statistic	Fitted distribution	Figure	KS value	P-value
10	A_n^2	LG(.5,1,2)	Fig (2-A1)	0.029	0.5855
	V_n^2	LN(1.5,3)	Fig(2-A7)	0.033	0.454
20	A_n^2	GGAM(0,4,1.5)	Fig (2-A2)	0.049	0.0713
	V_n^2	R(2,1.5)	Fig(2-A8)	0.045	0.1070
40	A_n^2	GGAM(1,3,1)	Fig (2-A3)	0.053	0.0586
	V_n^2	LG(1,2,.5)	Fig(2-A9)	0.049	0.0680
50	A_n^2	LG(1,1.5,1)	Fig (2-A4)	0.040	0.2102
	V_n^2	GL(.7,.7,.8)	Fig(2-A10)	0.028	0.6116
70	A_n^2	GL(0,2,.5)	Fig (2-A5)	0.026	0.7262
	V_n^2	GL(.9,1.5,2)	Fig(2-A11)	0.027	0.6566
100	A_n^2	LN(1,2)	Fig (2-A6)	0.023	0.8421
	V_n^2	LN(.8,.7)	Fig(2-A12)	0.031	0.5003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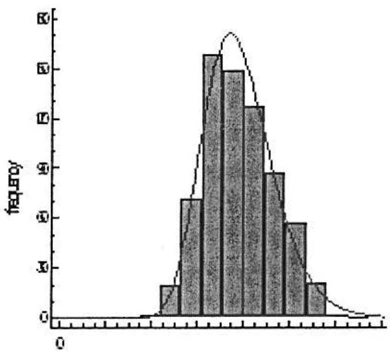
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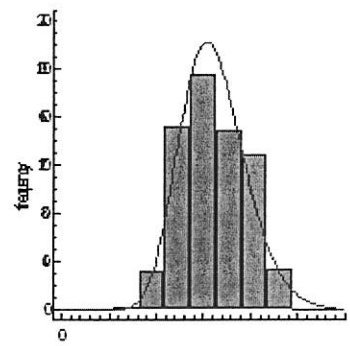
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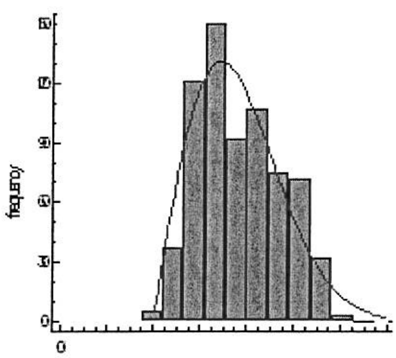
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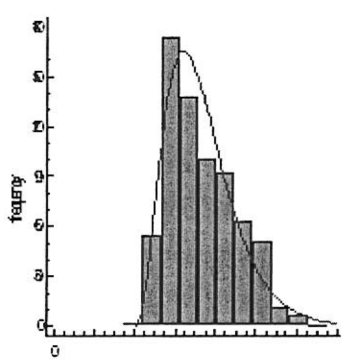
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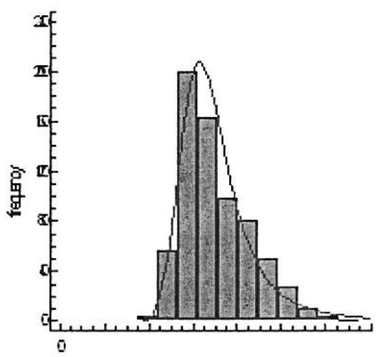
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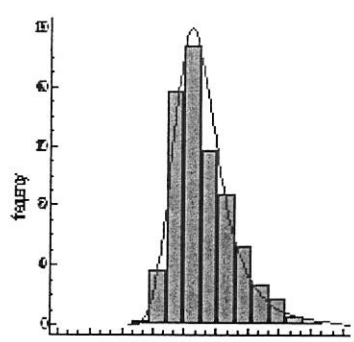

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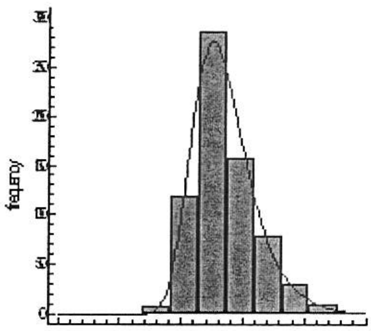

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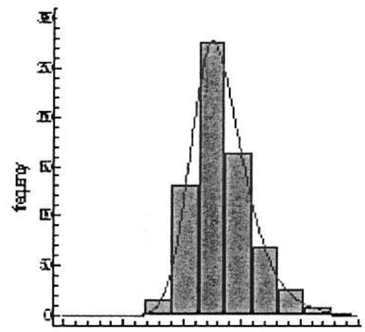

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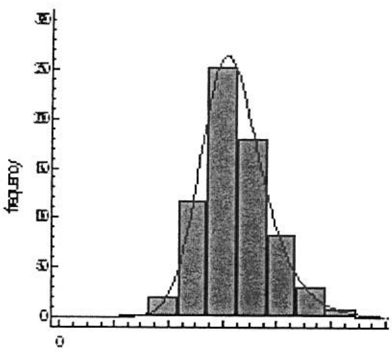

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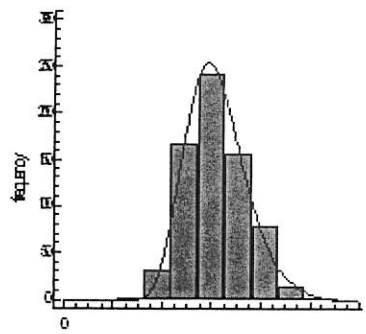

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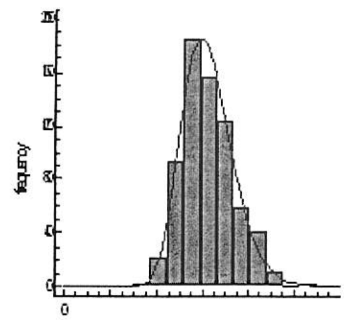

Fig (2-A12)

Figure (2) [A1-A12] Fitted Distribution

The sampling distributions of A_n^2 *and* V_n^2 using 10,000 simulated values are obtained for different values of sample size. We applied KS goodness of fit test to examine the sampling distribution of A_n^2 *and* V_n^2. We took 10,000 simulated values of A_n^2 *and* V_n^2 for each suggested values of sample size. Table (3) summarizes the results of this study and Figure (2) presents the histogram of the test statistics A_n^2 *and* V_n^2 and their sampling distributions. Therefore, we accept the corresponding sampling distribution of A_n^2 *and* V_n^2 for all cases.

The statistical package Statgraphics version 5 is used for this purpose. This package contains 45 probability distribution functions built in which are used to fit a collection of data and test the best fit using Kolomogorov-Smirnov (KS) test. By using the critical values for each test statistics, we draw the histogram for A_n^2 *or* V_n^2 and find the best fit using the Statgraphics.

Figure (2) [A1 - A12] contains the 12 histograms and the fitted distributions for each case of sample size and test statistics A_n^2 *or* V_n^2 displayed in Table (3).

The probability distribution functions for all fitted sampling distributions were:

Log logistic distribution with pdf, denoted by LG(μ,η,σ)

$$f(x)=\frac{1}{\sigma x}\frac{e^{z}}{(1+e^{z})^{2}},\ z=\frac{\ln(x-\mu)-\eta}{\sigma},x>\mu$$

Lognormal distribution with pdf, denoted by LN(μ,σ)

$$f(x)=\frac{1}{x\sqrt{2\pi}\sigma}e^{-\frac{(\ln(x)-\mu)^{2}}{2\sigma^{2}}},x>0$$

Generalized gamma distribution with pdf, denoted by GGAM(μ, n,λ)

$$f(x)=\frac{1}{\Gamma n}\lambda^{n}x^{n-1}e^{-\lambda(x-\mu)},x>\mu$$

Rayleigh distribution with pdf, denoted by R(μ,σ)

$$f(x)=\frac{2}{x-\mu}(\frac{x-\mu}{\sigma})^{2}e^{-(\frac{x-\mu}{\sigma})^{2}},x>\mu$$

Generalized logistic with pdf, denoted by GL(μ,σ,λ)

$$f(x)=\frac{\lambda}{\sigma}\frac{e^{-(\frac{x-\mu}{\sigma})}}{(1+e^{-(\frac{x-\mu}{\sigma})})^{1+\lambda}},x>\mu$$

5. Fitting the Real Life Data to Generalized Rayleigh Distribution

Wind energy is renewable and environment friendly. It is an alternative clear energy source compared to the fossil fuels that pollute the lower layer of atmosphere. The most important parameter of the wind energy is the wind speed. Statistical methods are useful for estimating wind speed because it is a random phenomenon. For this reason, wind speed probabilities can be estimated by using probability distributions. Wind energy is a form of solar energy; it is an air current created by the balance between pressure and temperature differences due to the different distribution of solar heat coming to earth.

In order to illustrate the use of the A_n^2 *and* V_n^2 critical values, regular and complete measurement data taken from the index Swedish Meteorological and Hydraulically Institute records for the daily wind speed. For the wind speed analysis undertaken the following procedures were followed:

(i) Organizing the daily readings in intervals and indicating how often a certain wind speed occurs in these intervals for 30 months during the years from 2006 to 2008; (ii) Computing

2.9, 4.7, 3.8, 3.4, 2.5, 3.3, 3.5, 2.6, 4.1, 3.3

6.9, 2.7, 2.0, 2.5, 2.8, 2.0, 3.2, 2.6, 3.8, 4.0

The analysis results of the test statistics used in determination of the effective distribution indicate that generalized Rayleigh distribution with shape parameter $\alpha = 1 \; and \; 2$ are accepted at 5% significance level according to $A_n^2 \; and \; V_n^2$. For this reason, the distribution best representing the set of data is the generalized Rayleigh distribution. The Rayleigh distribution is used in presence of a data set composed of mean wind speed (Pashardes and Chiristofides, (1995)).

6. Power study

The power of a goodness of fit test is defined as the probability that a statistic will lead to the rejection of the null hypothesis, H_0, when it is false, i.e. when a sample is not from the hypothesized population but an alternative population (Mann et al. (1974)), i.e. the power of the test is the frequency with which the null hypothesis is actually rejected at the given significance level when samples are drawn from the specified alternative distribution.

Monte Carlo simulation is applied to calculate the power of the two test statistics $A_n^2 \; and \; V_n^2$ at different sample size and two significance levels. Ten thousand samples are generated for each sample sizes $n = 10, 20, 40, 50, 70, \; and \; 100$ from each alternative distribution. Then the proportions of rejections are computed for 1% and 5% levels of significance. Samples are taken from the following four alternative distributions:

Gamma distribution with pdf $f(x) = \frac{1}{\Gamma(\theta)} x^{\theta - 1} e^{-x}$, $x > 0$, denoted by Gamma(θ)

Lognormal distribution with pdf $f(x) = \frac{1}{\theta x \sqrt{2\pi}} e^{\frac{-(\ln x)^2}{2\theta^2}}$, $x > 0$, denoted by LGN(θ)

Pareto distribution with CDF $F(x) = 1 - \frac{1}{(1+x)^{\theta}}$, $x > 0$, denoted by Pareto(θ)

Exponentiated Weibull with CDF $F(x) = (1 - e^{-x^{\theta}})^{\xi}$, $x > 0$, denoted by EW(θ, ξ)

From Table (4) it is evident that the power of the two tests is high against the generalized Weibull alternative; it is the highest among all the alternatives considered. Also, it should be noticed that the statistics have low power when the alternative is Pareto.

The V_n^2 statistic is generally has higher power compared to the A_n^2 statistic for all sample size and all alternative distributions selected at the two significance levels mentioned above.

Table (4)
Power Study Result

Sample size (n)	Alternative distribution	5% significance level		1% significance level	
		A_n^2	V_n^2	A_n^2	V_n^2
10	Gamma (2)	.117	.252	.027	.157
	LGN(1.5)	.293	.334	.216	.304
	Pareto(3)	.054	.163	.039	.099
	EW(1,2)	.564	.668	.433	.571
20	Gamma (2)	.133	.327	.032	.296
	LGN(1.5)	.452	.526	.376	.443
	Pareto(3)	.087	.255	.068	.145
	EW(1,2)	.656	.739	.521	.687
40	Gamma (2)	.245	.330	.077	.308
	LGN(1.5)	.576	.613	.499	.561
	Pareto(3)	.137	.331	.092	.195
	EW(1,2)	.743	.811	.613	.773
50	Gamma (2)	.433	.565	.156	.469
	LGN(1.5)	.662	.676	.523	.625
	Pareto(3)	.171	.387	.138	.260
	EW(1,2)	.766	.893	.692	.823
70	Gamma (2)	.639	.678	.296	.575
	LGN(1.5)	.710	.749	.649	.699
	Pareto(3)	.232	.422	.150	.332
	EW(1,2)	.811	.931	.741	.897
100	Gamma (2)	.781	.794	.465	.693
	LGN(1.5)	.733	.825	.707	.768
	Pareto(3)	.315	.472	.262	.422
	EW(1,2)	.892	.975	.819	.916

Entries are probability of rejecting H_0 when the random sample is actually from the stated alternatives distributions at two distinct levels of significance 1% and 5%

In this study, it is seen that the modified AD test V_n^2 has higher power while testing for

valuable comments for improving this paper.

References

Ahmad, K.E., Fakhry, M.E. and Jaheen, Z.F. (1997). Empirical Bayes estimation of P(Y < X) and characterization of Burr-type X model, Journal of Statistical Planning and Inference, 64, 297-308.

Anderson, T. W. and Darling, D. A. (1954). A test of goodness of fit, J. Amer. Statist. Assoc. 49, (1954), 765{769.

Choulakian, V. and Stephens, M. A. (2001). Goodness of fit tests for the generalized Pareto distribution. Technometrics, 43, 478-484.

D'Agostino, R. B. and Stephens, M. A. (1986). Goodness-of-Fit Techniques (Eds.), Marcel Dekker, New York, 1986.

Gupta, R. D. and Kundu, D. (2001). Exponentiated exponential family: an lternative to gamma and Weibull distributions. Biometrical J., 43, 117- 130.

Jaheen, Z.F. (1995). Bayesian approach to prediction with outliers from the Burr type X model, Microelectronic Reliability, 35, 45-47.

Jaheen. Z.F. (1996). Empirical Bayes estimation of the reliability and failure rate functions of the Burr type X failure model, Journal of Applied Statistical Sciences, 3, 281-288.

Kundu, D. and Raqab, M.Z. (2005). Generalized Rayleigh distribution: different methods of estimation, Computational Statistics and Data Analysis, 49, 187-200.

Kundu, D. and Gupta, R. D. (2007)A convenient way of generating gamma random variables using generalized exponential distribution. Comput. Statist. & Data Anal., 51, 6, 2796-2802.

Kundu, D. and Gupta, R. D. (2008).Generalized exponential distribution: Bayesian estimation, Comput. Statist.& Data Anal., 52, 4, 1873-1883.

Mann, N. R., Schafer, R. E. and Singpurwalla, N. D. (1974)" Methods for statistical analysis of reliability and life data." John Wiley & Sons.

Mudholkar, G. S. and Hutson, A.D. (1996). Exponentiated Weibull family: Some properties and flood data application. Comm. Statist. Theeor. Methods, 25, 3050-3083.

Pashardes, S., Christofides, C. (1995) Statistical analysis of wind speed and direction in and Statistical Sciences, 4(2), 179-193

Sartawi, H.A. and Abu-Salih, M.S. (1991). Bayes prediction bounds for the Burr type X model, Communications in Statistics - Theory and Methods, 20, 2307-2330

Sinclair, C. D., Spurr, B. D. and Ahmad, M. I. (1990). Modified Anderson Darling test. Commun. Statist. Theory and Meth., 19, 3677-3686.

Stephens, M. A. (1974). EDF statistics for goodness of fit and some comparisons. Journal of the American Statistical Association 69:730–737.

Stephnes, M. A. (1976)." Asymptotic power of EDF statistic for exponentially against gamma and Weibull alternatives. Tech. report No. 297, Dept. of Stat., Stanford University.

Stephnes, M. A. (1977)." Goodness-of-fit for the exterme-value distribution". Biometrika, 64, 583-588.

Surles, J.G. and Padgett, W.J. (1998). Inference for P(Y < X) in the Burr type X model, Journal of Applied Statistical Science, 7, 225-238.

Surles, J.G. and Padgett, W.J. (2001). Inference for reliability and stress-strength for a scaled Burr type X distribution, Lifetime Data Analysis, 7, 187-200.