Testing EBUCA Class of Life Distribution Using U-Test

N. ALghufily
Girls College of Education for Scientific Departments,
P. O. Box 27104, Riyadh 11417
Saudi Arabia

Abstract:

In this paper, a new class of life distribution namely exponential better than used in convex average and denoted by (EBUCA), or its dual (EWUCA) is introduced. Testing exponentiality against (EBUCA) based on U-statistics is investigated. The percentiles of these tests are tabulated for samples sizes n=5(1)40. The power of the test is simulated for some commonly used distributions in reliability. Pitman's asymptotic efficiency of the test is calculated and compared. Data for 40 patients suffering from blood cancer disease (Leukemia) is considered as a practical application of the proposed test on medical data.

Key Words: asymptotic normality, life distribution, positive ageing, U-statistic, power estimates, asymptotic relative efficiency, testing exponentiality, EBUCA.

1 Introduction

The aging life is usually characterized by a non-negative random variable $X \geq 0$ with distribution function (cdf) F and survival function (sf) $\overline{F} = 1 - F$. Associated with X is the notion of "random remaining life" at age t, denoted by X_t where X_t has an sf as

$$\overline{F}_t(x) = \frac{\overline{F}(x+t)}{\overline{F}(t)},\ x,t \geq 0. \tag{1.1}$$

Note that $X_t \overset{st}{\leq} X$, or $\overline{F}_t(x) \leq \overline{F}(x)$ (st denotes the stochastic ordering) if and only if $\overline{F}$ is an exponential distribution. Comparing X and X_t in various forms and types create classes of aging useful in many biomedical, engineering and statistical studies, see Barlow and Proschan (1981). It is well known that the relation $X_t \overset{st}{\leq} X$ or $\overline{F}_t(x) \leq \overline{F}(x)$ defines the class of new better than used (NBU). On the other hand, the relation $E(X_t) \leq E(X)$ defines the class of new better than used in expectation (NBUE), decreasing mean residual lifetime (DMRL), and exponential better than used (EBU). Testing exponentiality against anyone of these classes forms a vast literature pool. Most of the testing procedures are based on developing empirical estimates of departure from exponentiality in favor of the alternative class. The result test statistics are mainly version of U-statistics. For this vast literature we refer the reader to the surveys by Hollander and Proschan (1975) Doksum and Yandell(1984); Loh(1984); Hendi et al(1993) and Abu-

Yussef and Elsherpieny (2003). In the present paper, comparing between the life distribution of a new unit with that of the remaining life or a used unit in increasing convcx average order leads us to introduce a new class of life distribution. This new class is larger and perhaps more practical than the EBUC class introduced by Hendi and Algofily (2004). Our new class compares a new life to that is used (of age t) in a new or ordering sense which we call "increasing convex average" ordering.

The paper is organized as follows: Section 2 contains notations and basic properties which are used to introduce the class of the"exponential better than used in the increasing convex average" (denoted by EBUCA). In Section 3, we use U-statistic to test $H_0 : F \text{ is exponential}(\mu)$ versus $H_1 : F \in EBUCA \text{ and not exponential}$, where $\mu = \int_0^\infty \overline{F}(u)du$. Also, we simulate the critical points for the statistic used in the test through Monte Carlo methods for sample sizes n = 5(1)40. Next, in section 4 we calculate the power of the test based on some other alternative life distributions, including the linear failure rate, Gamma and Weibull distributions. To show the efficiency of our results, we calculate Pitman asymptotic efficiency and compare our result by this given by Kango (1993). Finally, we apply the test to real practical data given by Abouammah et al. (1994) in section 6.

On the other hand, an ordering of life variable that proved useful in producing classes of life distributions is due to Stoyan(1983), Bhattcharjee(1991) for definition and properties.

2 Definitions and properties

In this section we present definitions, notations and basic properties used throughout the paper. We also give the definition of the new better than used in the increasing convex average class of life distributions.

Let X and Y be non-negative random variables with distribution functions $F(x)$ and $G(x)$ respectively, and survival functions $\overline{F}(x)$ and $\overline{G}(x)$. We say that X is smaller than Y in :

(i) the usual stochastic order (denoted by $X \leq_{st} Y$), if $\overline{F}(x) \leq \overline{G}(x)$ for all X;

(ii) the increasing convex order (denoted by $X \leq_{icx} Y$), if

$$\int_x^\infty \overline{F}(u)du \leq \int_x^\infty \overline{G}(u)du \quad \text{for all } X;$$

(iii) the increasing convex average order (denoted by $X \leq_{icxa} Y$),

$$\int_0^\infty \int_x^\infty \overline{F}(u)dudx \leq \int_0^\infty \int_x^\infty \overline{G}(u)dudx \quad \text{for all } X.$$

See Ahmad et al.(2006), Deshpande et al.(1986) and Kaur et al (1994).

On the other hand, in reliability theory, it has been found useful to define non-parametric classes of lifetime distributions by stochastic comparison of survival function of the lifetime of a new one. For example, let $X_t = [X - t / x > t]$ denote the residual lifetime of X at time t, and it is the time to failure of a unit with lifetime t and let X be a non-negative random variable with distribution function F. We say that

(i) X (or F) is new better than used (denoted by $X \in NBU$) if $X \leq_{st} Y$ for all $t \geq 0$,

(ii) X (or F) is new better than used in the convex order (denoted by $X \in NBUC$) if $X \leq_{icx} Y$ for all $t \geq 0$;

The NBU was introduced by Bryson and Siddiqqui (1969) and independently by Marsharll and Proschan (1972). It has been extensively studied to become one of the most studied classes life distributions. The NBUC class is due to Cao and Wang (1991)l. And (EBU) was introduced by El-batal (2002).

Following the same ideas, we now introduce a new aging class of life distribution by stochastic comparison of the survival function of residual lifetime of a used unit with that of the lifetime of a new one in the increasing convex average order sense. In a formal way:

Definition 2.1: F belongs to EBUC iff

$$\int_{x+t}^{\infty} \overline{F}(u)du \le \mu \overline{F}(t)\, e^{-x/\mu} \quad , \qquad x,t>0 \tag{2.1}$$

Using the above definition to introduce a new class of life distribution namely exponential better than used in convex average order (EBUCA).

Definition 2.2: A life distribution F is said to be exponential better than used in convex average order (EBUCA) if

$$\int_{0}^{\infty}\int_{x+t}^{\infty} \overline{F}(u)du\, dx \le \mu \overline{F}(t) \int_{0}^{\infty} e^{-x/\mu}\, dx \quad , \qquad x,t>0 \tag{2.2}$$

The implications among IFR, IFRA, NBU, NBUC, EBU, EBUC, EBUCA, NBUE, and HNBUE classes of life distributions are:

$$\begin{array}{ccccccccc} IFR & \to & IFRA & \to & NBU & \to & NBUE & \to & HNBUE \\ & & & & \downarrow & & \uparrow & & \\ & & & & EBU & \to & EBUC & \to & EBUCA \end{array}$$

We propose a test statistic based on the U-statistics for testing $H_0 : F \text{ is exponential}(\mu)$; vs: H_1: F is exponential better than used in convex average order class of life distribution (EBUCA) and not exponential. In addition we use Monte Carlo method to compute the critical points for sample size n=5(1)40. The power of the test is also simulate for some commonly used distributions in reliability. Pitman's asymptotic efficiencies of the test statistic given with comparison with other procedures. Finally, an example using data from Abouammoh et al (1994) is used.

3 Testing Against the EBUCA Class

To test $H_0 : F \text{ is exponential}(\mu)$ against $H_1 : F \in EBUCA$ and not exponential we use the following measure of departure from H_0.

$$\delta_F = E\left[\mu^2 \overline{F}(t) - \int_0^{\infty} \frac{x^2}{2} dF(x+t)\right] \tag{3.1}$$

or

$$\begin{aligned} \delta_F &= \int_0^{\infty} [\mu^2 \overline{F}(t) - \int_t^{\infty} \frac{(u-t)^2}{2} dF(u)] dF(t) \\ &= \frac{1}{2}u^2 - \int_0^{\infty}\int_t^{\infty} \frac{(u-t)^2}{2} dF(u) dF(t) \end{aligned} \tag{3.2}$$

Note that $\delta_F = 0$ under H_0 and $\delta_F > 0$ under H_1. By using a random sample of size n, the empirical estimate δ_F is $\hat{\delta}_F$. Let $\bar{F}_n(x) = \frac{1}{n}\sum_{j=1}^{n} I(X_j > x)$ denote the empirical survival distribution, $dF_n(x) = \frac{1}{n}$, μ is estimated by the sample mean $\bar{X}$, then $\hat{\delta}_{F_n}$ is given by using (3.2) as:

$$\hat{\delta}_{F_n} = \int_0^\infty [\bar{X}^2 \frac{1}{n}\sum_{j=1}^{n} I(X_j > x) - \frac{1}{2n}\sum_{j=1}^{n}(X_j - x)^2 I(X_j > x)] dF_n(x) \tag{3.3}$$

i.e

$$\hat{\delta}_{F_n} = \frac{1}{2n^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}[X_k X_l - (X_j - X_i)^2 I(X_j > X_i)] \tag{3.4}$$

Where $I(x > y) = \begin{cases} 1 & x > y \\ 0 & otherwise \end{cases}$

Thus to make the test statistic $\hat{\delta}_F$ in (3.4) is scale invariant we take

$$\hat{\Delta}_{F_n} = \frac{\hat{\delta}_{F_n}}{\bar{X}^2} \tag{3.5}$$

Where $\bar{X} = \sum_{j=1}^{n} \frac{x_j}{n}$, is the usual sample mean.

Setting $\phi(X_1, X_2, X_3, X_4) = X_3 X_4 - (X_2 - X_1)^2 I(X_2 > X_1)$, and defining the symmetric kernel $\Psi(X_1, X_2, X_3, X_4) = \frac{1}{4!}\sum_R \phi(X_{i_1}, X_{i_2}, X_{i_3}, X_{i_4})$ Where the sum is overall arrangements of X_1,X_2,X_3 and X_4 then $\hat{\Delta}_{F_n}$ in (2.5) is equivalent to the U-statistic

$$U_n = \frac{1}{\binom{n}{4}}\sum_{i<j<k<l} \Psi(X_1, X_2, X_3, X_4) \tag{3.6}$$

The following theorem gives a summary of the large sample properties of $\hat{\Delta}_{F_n}$ or U_n.

Theorem 2.2

(i) As $n \to \infty, \sqrt{n}(u - \Delta_F)$ is asymptotically normal with mean 0 and variance is given by

$$\sigma^2 = Var[\mu^2 \bar{F}(X_1) - \frac{1}{2}\int_{X_1}^{\infty} u^2 f(u)du - \frac{1}{2}X_1^2 \bar{F}(X_1) + X_1 \int_{X_1}^{\infty} uf(u)du + \mu X_1$$
$$- \int_0^\infty \int_v^\infty u^2 f(u) f(v) du dv - \int_0^\infty v^2 \bar{F}(v) f(v) dv + 2\int_0^\infty \int_v^\infty vuf(u) f(v) du dv] \tag{3.7}$$

(ii) under H_0, the variance reduces to

$$\sigma^2 = Var[2X_1 - \frac{1}{2}X_1^2 - 1] = 1 \tag{3.8}$$

(iii) if F is continuous EBUCA, then test is consistent.

Proof

By using the standard theory of U-statistic [Lee(1990)], one can easily prove parts (i) and (ii) to prove part (iii) Let $D(x,t)=\mu^2\overline{F}(t)-\int_t^\infty(\frac{x-t}{2})^2dF$ in (3.2) and since F is EBUCA and continuous, $D(x,t)>0$ for at least one value of (x,t) call it (x_0,t_0). Now let $(x,t)=\inf\{(x,t)/(x>x_0 \text{ and } t>t_0)\}$, $\overline{F}(x)=\overline{F}(x_0)$ and $\overline{F}(t)=\overline{F}(t_0)$ Thus,

$$D(x,t)=\mu^2\overline{F}(t_1)-\int_{t_1}^\infty(\frac{x-t_1}{2})^2dF \geq \mu^2\overline{F}(t_1)-\int_{t_1}^\infty(\frac{x-t_0}{2})^2dF$$

$$=\mu^2\overline{F}(t_0)-\int_{t_0}^\infty(\frac{x-t_0}{2})^2dF=D(x_0,t_0)>0$$

And $F(x_1+\delta_1)-(F(x_1)>0$ and $F(t_1+\delta_2)-F(t_1)>0$ and since x_1 and t_1 are points of increases of F, thus $\Delta_{F_n}>0$ then the test is consistent. To use the above test, we calculate $\sqrt{n}\,\hat{\Delta}_{F_n}/\sigma_o$ and reject H_0 if it exceeds the normal vaniate $Z_{1-\alpha}$.

By using Monte Carlo methods, we calculate the empirical critical points of $\hat{\Delta}_{F_n}$ in (3.5) for different samples. Table (3.1) gives the lower and the upper percentile points for 1%,5%,10%90%,95% and 99%. The calculations are based on 1000 simulated samples sizes n=5(1)40.

Table (3.1): Critical Values for percentiles of $\hat{\Delta}_{F_n}$

N	1%	5%	10%	90%	95%	99%
5	-.73744	-.38227	-.24436	.29417	.32178	.36515
6	-.76339	-.36279	-.22952	.29060	.32426	.36561
7	-.76365	-.33432	-.20354	.27870	.30960	.36247
8	-.62843	-.36945	-.21896	.28112	.30479	.36713
9	-.71701	-.39820	-.23580	.26260	.29294	.35024
10	-.72191	-.37831	-.25455	.25737	.28729	.35401
11	-.66843	-.36067	-.24209	.24240	.28281	.33636
12	-.67945	-.30511	-.21051	.26111	.29213	.33135
13	-.74705	-.41252	-.24820	.24672	.27410	.32883
14	-.70292	-.34191	-.21569	.23684	.27092	.31404
15	-.67595	-.34895	-.22733	.23456	.26512	.33165
16	-.59832	-.36606	-.24983	.23079	.26395	.30975
17	-.66002	-.35772	-.23658	.23003	.26707	.32508
18	-.62885	-.32886	-.21700	.22886	.26035	.32034
19	-.63041	-.33618	-.21470	.22060	.24823	.30731
20	-.68709	-.35137	-.21663	.20706	.23559	.28596
21	-.62186	-.30824	-.20521	.21499	.24577	.29861
22	-.56914	-.30723	-.20836	.20756	.24599	.28895
23	-.62139	-.30715	-.20870	.20338	.24314	.28819
24	-.57148	-.33255	-.22418	.19191	.22493	.28387
25	-.69867	-.29831	-.19967	.19829	.22832	.28751
26	-.57855	-.29736	-.19600	.18944	.22082	.26543
27	-.57609	-.31540	-.19664	.19123	.21887	.27755
28	-.47441	-.28937	-.20370	.18709	.21780	.25776
29	-.55312	-.28170	-.19398	.19038	.21637	.27797
30	-.44403	-.26430	-.17968	.19247	.22016	.29233
31	-.51922	-.27731	-.19344	.18833	.21770	.26959
32	-.48681	-.27831	-.18624	.18814	.21347	.25728
33	-.57383	-.29343	-.21183	.17715	.21002	.24710
34	-.48793	-.27135	-.18889	.17454	.20558	.25059
35	-.51067	-.28637	-.17935	.17109	.20373	.26904
36	-.48326	-.27843	-.17493	.17928	.21764	.27439
37	-.40224	-.23927	-.16078	.17634	.20027	.25285
38	-.52726	-.25409	-.17594	.16800	.19994	.24455
39	-.40285	-.26104	-.18274	.16717	.19787	.23398
40	-.46174	-.24528	-.17690	.17217	.19491	.24053

4 The Power Estimates

The power estimate of the test statistic $\hat{\Delta}_{F_n}$ in (3.5) is considered for the significant level at 95% upper percentile and commonly used distributions in reliability modeling. These distributions are:

i) Linear Failure Rate: $\overline{F}_1(x) = \exp(-x - \frac{1}{2}\theta x^2)\ ;\theta > 0, x \geq 0.$

(ii) Makeham : $\overline{F}_2(x) = \exp(-x - \theta(x + e^{-x} - 1))\ ;\theta > 0, x \geq 0.$

(iii) Weibull : $\overline{F}_3(x) = \exp(-x^{\theta})\ ;\theta > 0, x \geq 0.$

(iv) Gamma : $\overline{F}_4(x) = \int_x^{\infty} u^{\theta-1}\exp(-u)du / \Gamma(\theta)\ ;\theta > 0, x \geq 0.$

All these distributions have increasing failure rate (IFR) (for an appropriate restriction on θ), hence they all belong to a wider class. Moreover, all these distributions are reduced to exponential distribution for appropriate values of θ. Table (4.1) contains the power estimate for $\hat{\Delta}_{F_n}$ test statistic with respect to these distributions. The estimates are based on 1000 simulated samples of sizes n=10, 20 and 30 at significant level 95% upper percentile.

Table (4.1) Power estimates for $\hat{\Delta}_{F_n}$ -Statistic

Distribution	θ	Sammpl size		
		n=10	n=20	n=30
F_1: Linear Failure rate	2	0.27100	0.50400	0.59700
	3	0.32200	0.58600	0.71100
	4	0.35700	0.65400	0.77900
F_2: Makeham	2	0.08600	0.17500	0.14900
	3	0.11800	0.15000	0.18700
	4	0.14100	0.18900	0.19900
F_3: Weibull	2	0.75200	0.97700	0.99800
	3	0.99700	1.00000	1.00000
	4	1.00000	1.00000	1.00000
F_4: Gamma	2	0.36300	0.64600	0.68900
	3	0.68000	0.92500	0.97100
	4	0.85400	0.98700	0.99800

From Table (4.1), we can see that the power values are almostly increasing as both θ and n increasing.

5. Pitman asymptotic efficiency

The "Pitman asymptotic efficiency" (PAE) of any test $\hat{\Delta}_{F_n}$ is given by

$$PAE\left[\Delta_{F_n}(\theta)\right] = \frac{1}{\sigma_0}\left[\frac{\partial}{\partial\theta}\Delta_{F_n}\Big|_{\theta=\theta_0}\right] \tag{5.1}$$

Since the above test $\hat{\Delta}_{F_n}$ in (3.5) is new and no other tests are known for EBUCA class, then we obtained the (PAE) of our test as follows:

$$PAE[\Delta_{F_n}(\theta)] = \frac{1}{\mu_{\theta_0}^2} \int_0^\infty \left[\mu_{\theta_0}^2 \overline{F}'_{\theta_0}(t) + \overline{F}_{\theta_0}(t) 2\mu_{\theta_0}\mu'_{\theta_0} - \int_t^\infty \frac{(u-t)^2}{2} f'_{\theta_0}(t)du \right] f_{\theta_0}(t)dt \quad (5.2)$$

We compare our test to other classes. Here we choose the test U_n presented by Kanjo (1993) for NBUE class. The comparison is achieved by using Pitman asymptotic relative efficiency (PARE), which can be defined as follows:

Let T_{1n} and T_{2n} be two test statistics for testing $H_0 : F_\theta \in \{F_{\theta_n}\}\ \theta_n = \theta + cn^{-1}$, where c an arbitrary constant, then the asymptotic relative efficiency of T_{1n} relative to T_{2n} can be defined as:

$$e(T_{1n}, T_{2n}) = \frac{\{\mu'_1(\theta_0)/\sigma_1(\theta_0)\}}{\{\mu'_2(\theta_0)/\sigma_2(\theta_0)\}}.$$

Where $\mu'_i(\theta_0) = \left\{ \lim_{\theta \to \infty} [\frac{\partial}{\partial \theta} E(T_{in})] \right\}_{\theta \to \theta_0}$ and $\sigma_i(\theta_0) = \lim_{n \to \infty} Var(T_{in}), i = 1,2$ is the null variance.

Table (5.1) contains PAE's of (i) the linear failure rate family, (ii) Makeham family and (iii) Weibull family for $\theta > 0$ as alternatives, by using (5.2).

Note that H_0 (the exponential) is attained at $\theta = 0$ in (i) and (ii), and attained at $\theta = 1$ in (iii). Also we give PARE of $\hat{\Delta}_{F_n}$ and U_n tests.

Table (5.1): Efficiencies (PAE) of $\hat{\Delta}_{F_n}$ and U_n.

Distribution	Efficiency (PAE) of		$e(\hat{\Delta}_{F_n}, U_n)$
	$\hat{\Delta}_{F_n}$	Y_n	
F_1: Linear Failure rate	1.000	0.433	2.310
F_2: Makeham	0.25	0.144	1.736
F_3: Weibull	1.00	0.132	7.576

The efficiencies in Table (5.1), show clearly the procedure $\hat{\Delta}_{F_n}$ of U-test method performs well for F_1, F_2 and F_3 than the procedure U_n of Kango (1993) and more efficient.

6 Application

In this section, we calculate the $\hat{\Delta}_{F_n}$ test statistic for the data set of 40 patients suffering from blood cancer (Leukemia) from one of the Ministry of Health Hospitals in Saudi Arabia [see Abouammah et al. (1994)]. The ordered life times (in days) are: 115, 181, 255, 418, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1024, 1062, 1063, 1165, 1191, 1222, 1222, 1251, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1578,1599, 1603, 1605, 1696, 1735, 1799, 1815, 1852.

It was found that the test statistics $\hat{\Delta}_{F_n}$ in (3.5) has value $\hat{\Delta}_{F_n} = 0.3988$ which is greater than the critical value in Table (3.1) at 95% upper percentile. We therefore accept H_1 which states that the data has EBUCA property.

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